INSTRUMENTATION DEVICES AND SYSTEMS-I

SUBJECT CODE: PCEI4302

B.Tech, Fourth Semester

Prepared By

Dr. Kanhu Charan Bhuyan

Asst. Professor

Instrumentation and Electronics Engineering

COLLEGE OF ENGINEERING AND TECHNOLOGY

BHUBANESWAR

INSTRUMENTATION DEVICES AND SYSTEMS-I

Module-1

Elements of Measurement System;

Static Characteristics, Systematic Characteristics, Statistical Characteristics, calibration

Dynamic Characteristics, transfer functions of typical sensing elements, step and frequency response of first and second order elements, dynamic error in measurement systems

Module-2

Sensing Elements:

Resistive sensing elements: potentiometers, Resistance Temperature Detector (RTD), thermistors, strain gages.

Capacitive sensing elements: variable separation, area and dielectric;

Inductive sensing elements: variable reluctance and LVDT displacement sensors;

Electromagnetic sensing elements: velocity sensors,

Thermoelectric sensing elements: laws, thermocouple characteristics, installation problems, cold junction compensation.

IC temperature sensor

Elastic sensing elements: Bourdon tube, bellows and diaphragms for pressure sensing, force and torque measurement.

Module-3

Signal Conditioning Elements:

Deflection bridge: design of resistive and reactive bridges, push-pull configuration for improvement of linearity and sensitivity

Amplifiers: Operational amplifiers-ideal and non-ideal performances, inverting, non-inverting and differential amplifiers, instrumentation amplifier, filters, A.C. carrier systems, phase sensitivity demodulators and its applications in instrumentation

Flow Measurement:

Basics of flow measurement: differential pressure flowmeters- Pitot tube, Orifice plate, Ventri tube; Rotameter, turbine type flowmeter, electromagnetic flowmeter, Doppler shift flowmeter.

Text Books:

1. Principle of Measurement Systems- J.P. Bentley (3/e), Pearson Education, New Delhi, 2007.
2. Introduction to Measurement and Instrumentation- A. K. Ghosh(3/e), PHI Learning, New Delhi, 2009

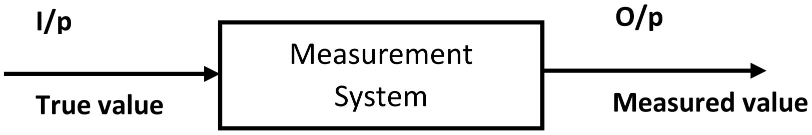
3. Transducers and Instrumentation- D. V. S. Murthy(2/e), PHI Learning, New Delhi, 2009.

Reference Books:

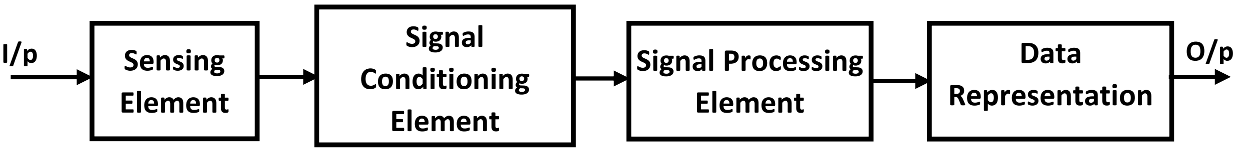
1. Measurement Systems Application and Design- E. O. Doeblin (4/e), McGraw-Hill, International, NY.
2. Instrumentation for Engineering Measurements- J.W. Dally, W.F. Riley and K.G. McConnel (2/e), John Wiley, NY, 2003.
3. Industrial Instrumentation- T.R. Padmanabhan, Springer, London, 2000

Module-I

1. Elements of Measurement System:



Block Diagram of Measurement System



1. **Sensing Element**

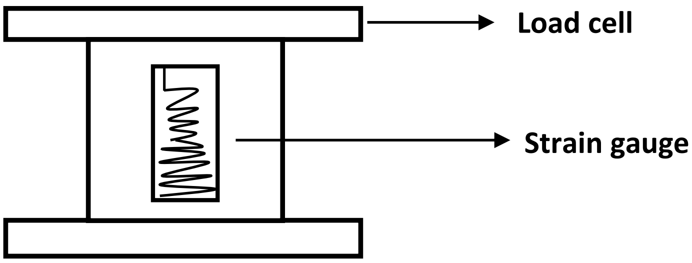
   Sensing elements take the input and give the output which depends on the variable to be measured. It consists of two parts.

   I. Primary Sensing Element
   II. Secondary Sensing Element

- **Sensor:** - Sensor is a primary sensing element which converts one form of physical quantity to another form.
- **Transducer:** - It is a device which convert one form of energy to another form.
  In electrically form generally transducer convert mechanical energy to electrical energy.

[Ques. Difference between Sensor and Transducer?]

Ex: - Strain gauge mounted on the load cell. Load cell is the primary Sensing element.



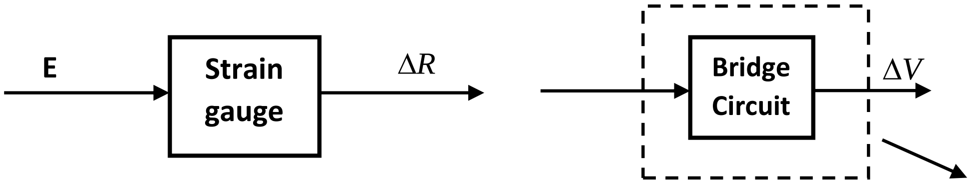
I/p- Strain O/p- Resistance change finally voltage

Ex: Thermocouple.

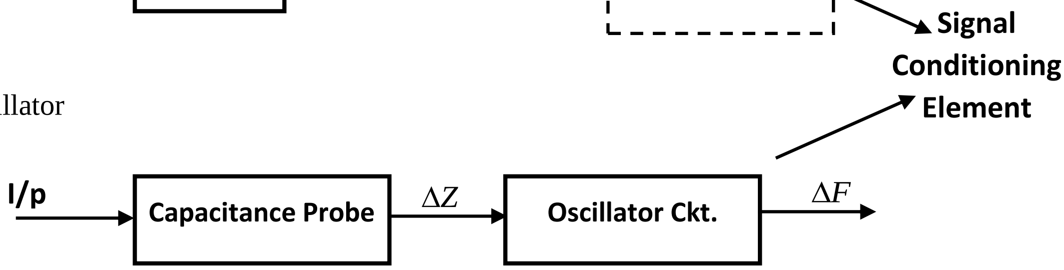
2. Signal Conditioning Element

The input to signal conditioning element is from the sensing element and it gives an output that I more suitable for further processing. The Output is generally in the form of DC voltage, DC current and frequency.

Ex1: Bridge circuit



Ex2: Oscillator



3. Signal Processing element

It takes the output of signal conditioning element as an input and gives an output which is suitable for the presentation.

Ex: 1. Analog to digital converter (A-D-C)

2. Micro Computer

4. Data Representation

It takes the output of signal processing element as an input and gives an output which is easily observed/measured by the observer.

Ex: 1. Pointer scale indicator

2. Alpha Numeric Display

3. Visual Display

Ex. of Measurement system:

Weight Measuring System,

Sensing Element,

Bridge Circuit: Change in Resistance to electrical form,

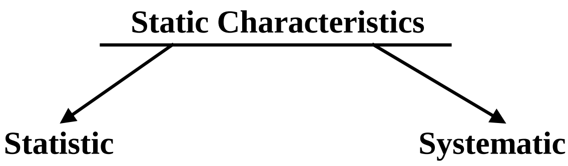
Amplifier mV-V,

A-D-C,

Micro Computer visual Display,

Visual Display.

2. Static Characteristics



Static Characteristics means the input is not going to be changed or, changed very shortly.

Statistical Characteristics: those characteristics which can be exactly quantify by any particular element.

Ex: Repeatability

Tolerance level

Systematic Characteristics: Those characteristics which can be exactly quantified by a particular element.

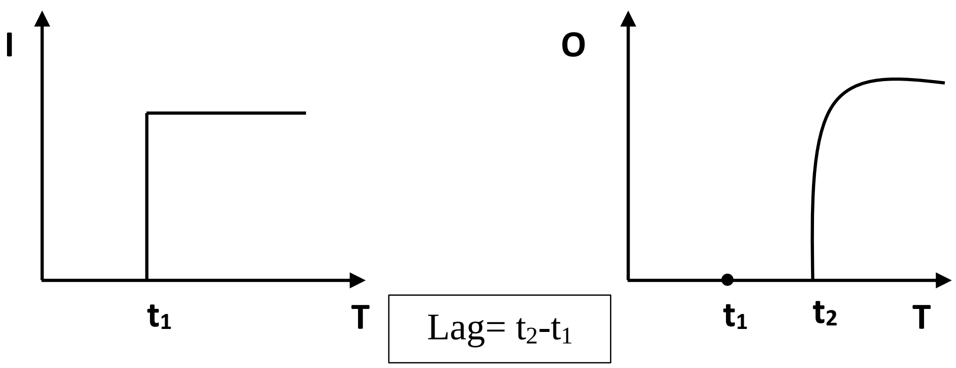
1. Dead Space
2. Threshold
3. Lag
4. Range
5. Span
6. Linearity
7. Non-Linearity
8. Hysteresis
9. Resolution
10. Sensitivity
11. Wear and ageing
12. Error Band

Systematic Characteristics:

1. *Dead Space:* It is the range of input for which there is no change in output.

2. *Threshold*: If dead space occurs at the beginning of the measurement then that is called threshold.

3. *Lag*: It is the time taken by an element or, Instrument to respond to an activity.



4. *Range:*

For minimum input range I_{MIN} For Minimum output range O_{MIN}

For Maximum input range I_{MAX} For Maximum output range O_{MAX}

5. *Span:* Span is the maximum variation between input and output.

Input Span= $I_{span}= I_{MAX}-I_{MIN}$

Output Span= $O_{span}= O_{MAX}-O_{MIN}$

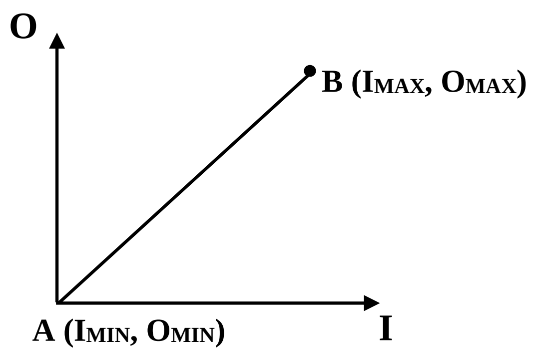
6. *Linearity:* The variation of input and output makes a straight line.

$$O - O_{MIN} = \left(\frac{O_{MAX} - O_{MIN}}{I_{MAX} - I_{MIN}} \right) . (I - I_{MIN})$$

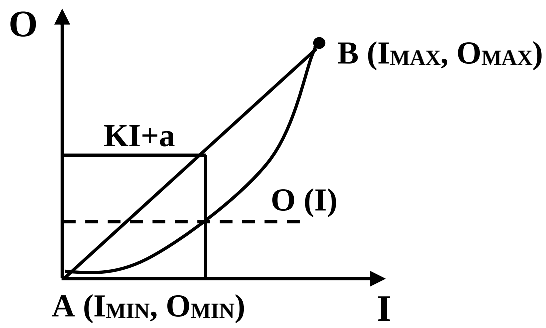
$O_{IDEAL} = KI + a$

$$K = \frac{O_{MAX} - O_{MIN}}{I_{MAX} - I_{MIN}}$$

$$\Rightarrow a = O_{MIN} - KI_{MIN}$$



7. *Non-linearity:* It is the difference between linear line and the actual line.



$$N(I) = O_{actual} - O_{linear}$$

$$O(I) = N(I) + KI + a$$

$$\hat{N} \rightarrow Max\ Non-linearity\ in\ FSD$$

Percentage of Full Scale Deflection (FSD)

$$\hat{N} = \frac{\hat{N}}{O_{MAX} - O_{MIN}} \times 100\%$$

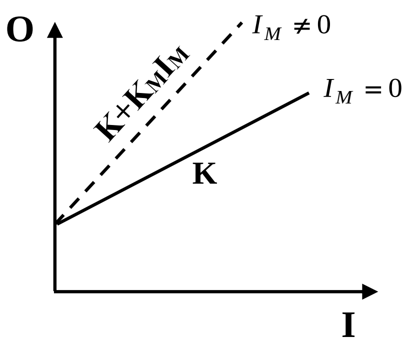
8. *Sensitivity:* The rate of change of output with respect to input.

$$\frac{dO}{dI} = K \rightarrow Linearity$$

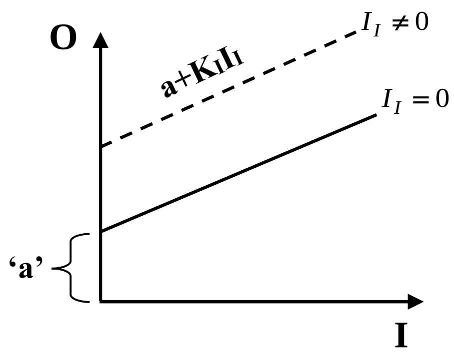
$$\frac{dO}{dI} = K + \frac{dN(I)}{dt} \rightarrow Non-Linearity$$

Environmental Effect

1. Modifying input (I_M) Changes the sensitivity or, Slope k
2. Interfering input (I_I) Changes the intercept 'a'.



(Modifying input)



$$O(I) = N(I) + KI + a$$

$$O(I) = N(I) + (K + K_M I_M)I + a + K_I I_I$$

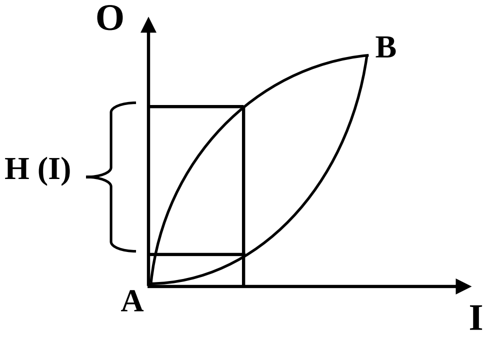
$$\Rightarrow O(I) = N(I) + KI + K_M I_M I + a + K_I I_I$$

K_M and K_I are environmental Coupling constants.

9. *Hysteresis:* For the given value of input the output may be different whether the input is increasing or, decreasing.

$$H(I) = O(I)_{I\downarrow} - O(I)\uparrow$$

$$\hat{H} = \frac{H(I)}{O_{Max} - O_{Min}}$$

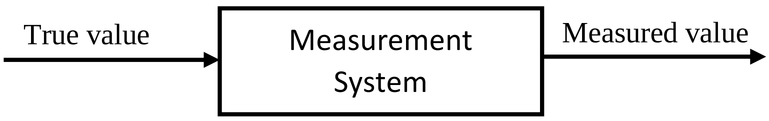


Ex: Backlash: Conversion of rotational movement to linear movement

10. *Resolution:* It is defined as the largest change in the input that can occur without any corresponding change in output.

Ex: Potentiometer resistance change with respect to distance.

11. *Accuracy:*



A= Measured Value – True value

12 *Wear and ageing:* These effects can cause the characteristic of an element. E.g. 'K' and 'a' to change slowly but systematically throughout its life.

Ex: Stiffness of the spring

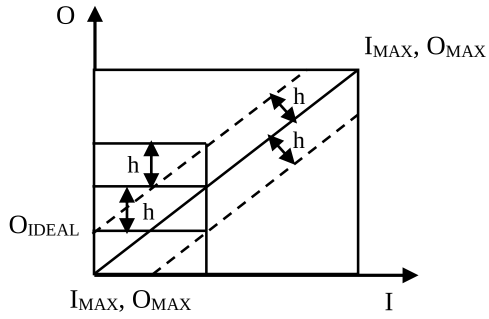
$K(t) = K_0 - bt$

K_0→ initial Stiffness

b→ constant,

$K(t)$→Stiffness of the spring.

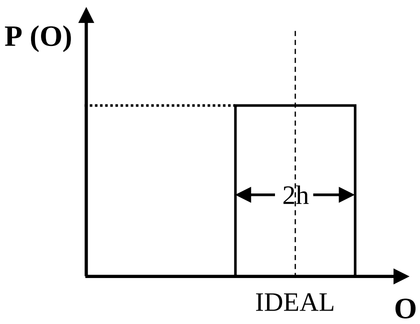
13 *Error Band:*



$\pm h$

P (O) = Probability Function

In rectangular form

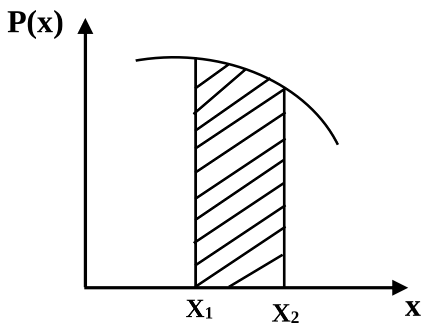


$$
\begin{aligned}
P(O) &= 1, \quad O_{IDEAL} - h \leq 0 \leq O_{IDEAL} + h \\
&= 0, \quad 0 \leq O_{IDEAL} - h \\
&= 0, \quad 0 \geq O_{IDEAL} + h
\end{aligned}
$$

In Gaussian form

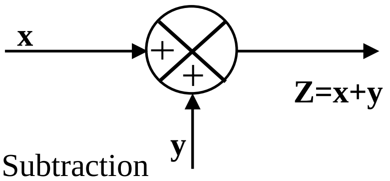
$$P(O)_{x_1, x_2} = \int_{x_1}^{x_2} P(x)\, dx$$

P(x)→ Probability density function

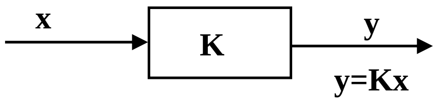


Symbols

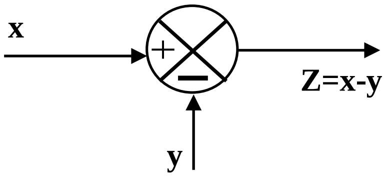
Addition



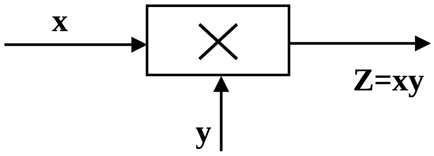
Scalar Multiplication



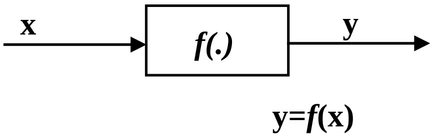
Subtraction



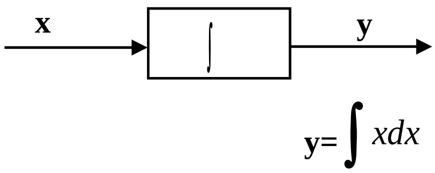
Variable Multiplication



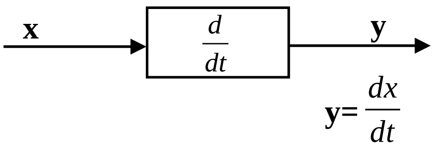
Function



Integration

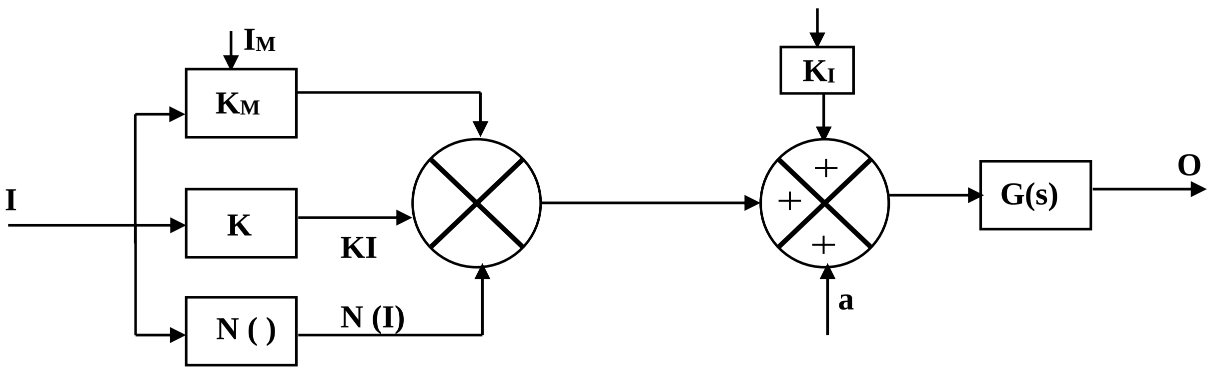


Differentiation



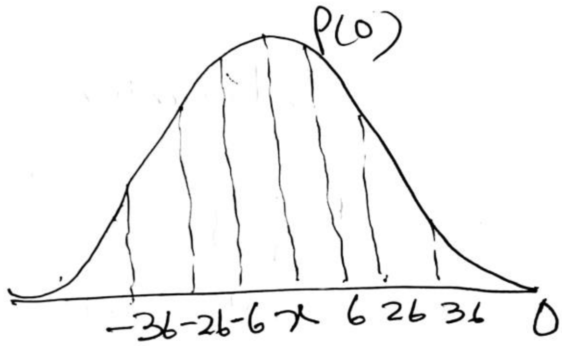
Generalised Model of system element.

$$O = KI + a + N(I) + K_M I_M I + K_I I_I$$



Statistical Characteristics:

1. *Repeatability:* It is the property of an element to repeat itself give same output for same input when the input is applied.



To centralise the mean value the Gaussian density function is used

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\bar{x})^2}{2\sigma^2}\right]$$

Where,

$$\sigma \rightarrow \text{Standard deviation}$$

$$\bar{x} \rightarrow \textit{Mean Value}$$

$$\bar{x} = \int_{-\infty}^{\infty} xP(x)dx$$

$$Variance = \sigma^2$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\bar{x})P(x)dx$$

$$O = KI + a + N(I) + K_M I_M I + K_I I_I$$

$$\Delta O = \left(\frac{\partial O}{\partial I}\right)\Delta I + \left(\frac{\partial O}{\partial I_M}\right)\Delta I_M + \left(\frac{\partial O}{\partial I_M}\right)\Delta I_I$$

$$y = a_1x_1 + a_2x_2 + a_3x_3$$

$$\sigma = \sqrt{\left(a_1{}^2\sigma_1^2 + a_2{}^2\sigma_2^2 + a_3{}^2\sigma_3^2\right)}$$

Standard deviation of output for a single element

$$\sigma_o = \sqrt{\left(\frac{\partial_O}{\partial_I}\right)^2 \sigma_I{}^2 + \left(\frac{\partial_O}{\partial_{I_M}}\right)^2 \sigma_{I_M}{}^2 + \left(\frac{\partial_O}{\partial_{I_I}}\right)^2 \sigma_{I_I}{}^2}$$

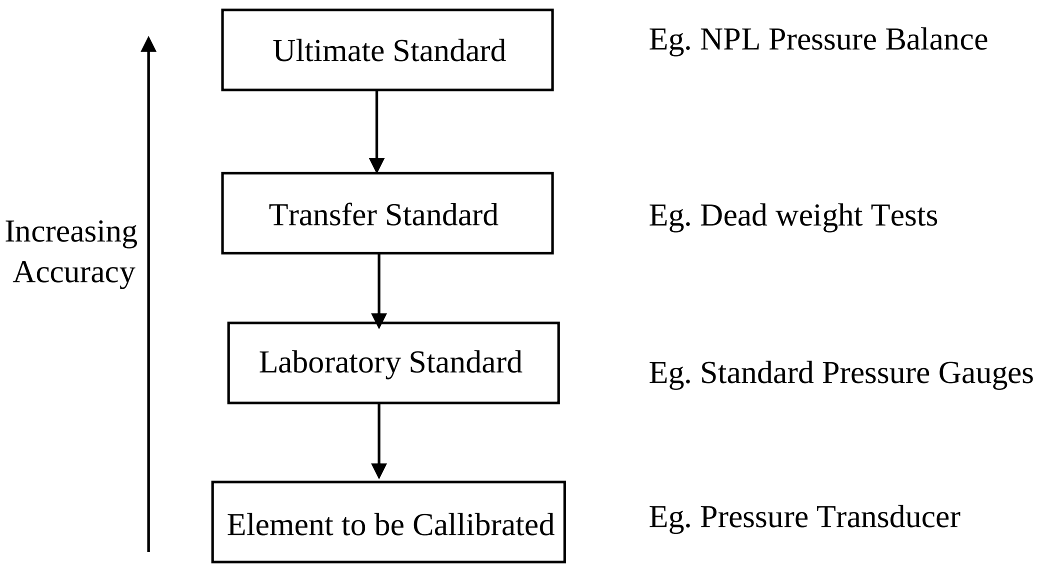
The Mean value of output or a single element is

$$\overline{O} = K\bar{I} + a + N(\bar{I}) + K_M \bar{I}_M \bar{I} + K_I \bar{I}_I$$

Identification of Static Characteristics:

Callibration: -

- The static characteristics of an element can be found experimentally by measuring corresponding values of the input 'I', the output 'O' and the environmental input (I_M, I_I). When I either at a constant value or changing slowly. This type of experiment is referred to as Callibration
- The instruments and techniques used to quantify the variables I, O, I_M and I_I are referred to as standard.
- The true value of a variable is the measured value obtained with a standard of ultimate accuracy.



NPL→ National Physical Laboratory

There are three steps to measure a parameter:

(i) O vs. I $\Big|_{I_M = I_I = 0}$

(ii) O vs. I_M, I_I $\Big|_{I = \text{Constant}}$

$I = (I_{MIN} + I_{MAX})/2$

$$K_M = \frac{\Delta O}{\Delta I_M} = \left(\frac{2}{I_{MIN} + I_{MAX}}\right)$$

$$O = KI + a + N(I) + K_M I_M I + K_I I_I \qquad (1)$$

$$O + \Delta O = KI + a + N(I) + K_M (I_M + \Delta I_M) I + K_I I_I \qquad (2)$$

Subtract (1) from (2)

$$\Delta O = K_M \Delta I_M I$$

$$K_M = \frac{\Delta O}{\Delta I_M I} \Rightarrow K_M = \frac{\Delta O}{\Delta I_M}\left(\frac{2}{I_{MIN} + I_{MAX}}\right)$$

Interfering I/P with known value of K_I

$$O = KI + a + N(I) + K_M I_M I + K_I I_I \qquad (1)$$

$$O + \Delta O = KI + a + N(I) + K_M (I_M + \Delta I_{M,I}) I + K_I (I_I + \Delta I_{I,M}) \qquad (2)$$

Subtracting (1) from (2)

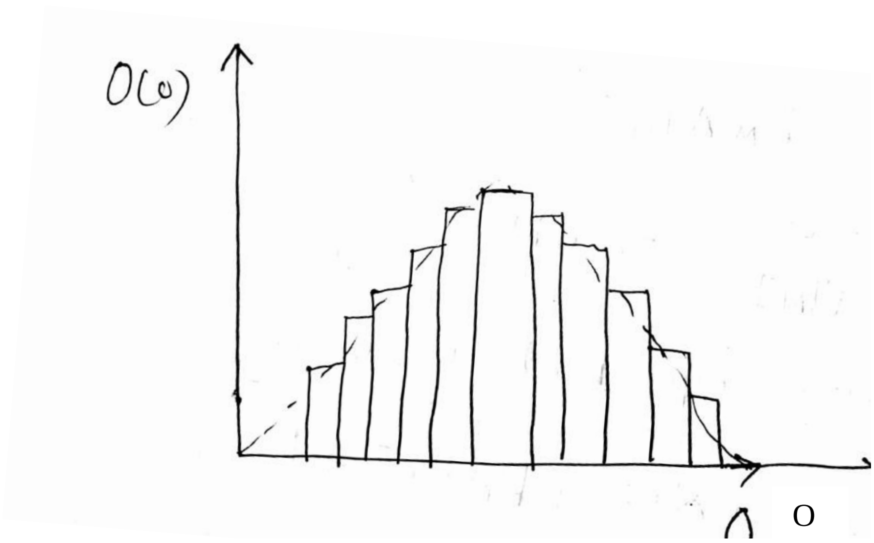
$$\Delta O = K_M \Delta I_{M,I} I + K_I \Delta I_{I,M}$$

$$\Rightarrow K_M \Delta I_{M,I} I = \Delta O - K_I \Delta I_{I,M}$$

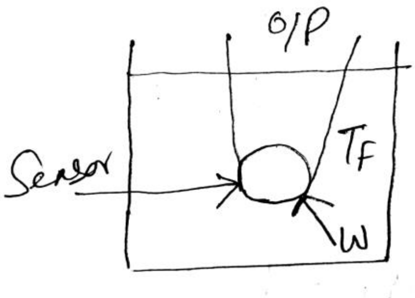
$$\Rightarrow K_M = \frac{\Delta O - K_I \Delta I_{I,M}}{\Delta I_{M,I} I}$$

$$\Rightarrow K_M = \left(\frac{\Delta O}{\Delta I_{M,I}} - K_I\right)\left(\frac{2}{I_{MIN} + I_{MAX}}\right)$$

(iii) *Repeatability test*



3. Dynamic Characteristics:



When $T_F > T$

Outflow=U

Inflow (W) =UA (T_F - T)

W= $MC\frac{dT}{dt}$

dT→ Change in temp. C→ Specific heat

W→Inflow dt→ Change in time

Characteristics of sensor is defined by **heat balance equation.**

Rate of heat flow to sensor **-** Rate of heat outflow **=** Rate of change of sensor heat content

U= Overall heat transfer co-efficient

A= Effective area of sensor

M=Mass of sensor

C=Specific heat of sensor material

T_F = Temperature of Fluid

T= Temperature of sensor

Unit

U= watt/m^2 ·c

C=J/kg ·c

$$\begin{aligned} when\ t<0,\ T_F &= T \\ t=0,\ T_F &\to T_F+\Delta T_F \\ T &\to T+\Delta T \end{aligned}$$

$$W = UA(T_F + \Delta T_F - T - \Delta T)$$

$$MC\frac{d(T+\Delta T)}{dt} = UA(T_F + \Delta T_F - T - \Delta T)$$

$$\Rightarrow UA(T_F - T) + UA\Delta T_F - UA\Delta T = MC\frac{dT}{dt} + MC\frac{d\Delta T}{dt}$$

$$\Rightarrow UA\Delta T_F - UA\Delta T = MC\frac{d(\Delta T)}{dt}$$

$$\Rightarrow UA(\Delta T_F - \Delta T) = MC\frac{d(\Delta T)}{dt}$$

$$\Rightarrow \Delta T_F - \Delta T = \frac{MC}{UA}\frac{d(\Delta T)}{dt}$$

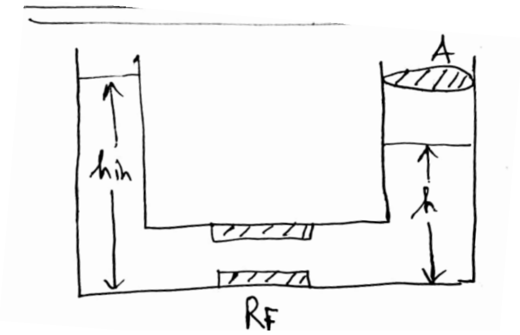
$$\Rightarrow \frac{MC}{UA}\frac{d(\Delta T)}{dt} + \Delta T = \Delta T_F$$ ➔ Linear 1st order differential equation for thermal system

$$\frac{MC}{UA} = \tau(\text{time constant})$$

$$\Rightarrow \tau.\frac{d(\Delta T)}{dt} + \Delta T = \Delta T_F$$

Units $$\frac{MC}{UA} = \frac{{}^{k}\!/\!_{g}\; J/{}^{k}\!/\!_{g}\;{}^{\circ}C}{{}^{w}\!/\!_{m^2\;{}^{\circ}C} \times m^2} = {}^{J}\!/\!_{w} = \sec$$

For Fluidic System



$R_F \rightarrow$ Resistance of fluid

$$P_{in} = \rho g h_{in} \quad (\because P = \rho g h)$$

$$P_{in} - P = \rho g(h_{in} - h)$$

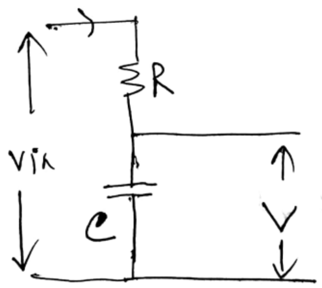
$$\frac{P_{in} - P}{R_F} = \frac{Adh}{dt} \Rightarrow \frac{\rho g(h_{in} - h)}{R_F} = \frac{Adh}{dt}$$

$$\Rightarrow R_F.\frac{Adh}{dt} + \rho g h = \rho g h_{in}$$

$$\Rightarrow \frac{AR_F}{\rho g}.\frac{dh}{dt} + h = h_{in}$$ ➔ 1st order differential equation for thermal system

$$\tau = \frac{AR_F}{\rho g} = \text{time constant}$$

For Electrical System



$$V_{in} - IR - V = 0$$

$$C = \frac{Q}{V} \Rightarrow Q = CV$$

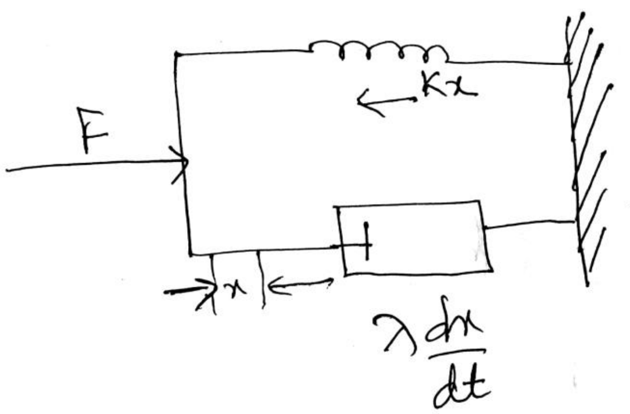
$$V_{in} - V = \frac{dQ}{dt} R$$

$$\Rightarrow V_{in} - V = RC \frac{dV}{dt}$$

$$\Rightarrow RC \frac{dV}{dt} + V = V_{in}$$

$$\text{Time Constant}(\tau) = RC$$

For Mechanical System



$$\lambda \frac{dx}{dt}$$

$\lambda = damping\ \ co - efficient$

$k = Stiffness\ of\ spring$

$$F = kx$$

$$\Rightarrow F - kx = \lambda \frac{dx}{dt} \text{(when there is a change in displacement)}$$

$$F + \Delta F - k(x + \Delta x) = \frac{\lambda d(x + \Delta x)}{dt}$$

$$\Rightarrow F - kx + \Delta F - k\Delta x = \lambda \frac{dx}{dt} + \frac{\lambda d(\Delta x)}{dt}$$

$$\Delta F - k\Delta x = \frac{\lambda d(\Delta x)}{dt}$$

$$\Rightarrow \frac{\lambda d(\Delta x)}{dt} + k\Delta x = \Delta F$$

$$\Rightarrow \frac{\lambda}{k} \frac{d(\Delta x)}{dt} + \Delta x = \frac{\Delta F}{k}$$

$$\tau = \frac{\lambda}{k} = \text{timeconstant}$$

Property	Thermal	Electrical	Fluidic	Mechanical
τ	$\frac{MC}{UA}$	RC	$\frac{AR_F}{\rho g}$	$\frac{\lambda}{k}$
R	$\frac{1}{UA}$	R	R_F	λ
C	MC	C	$\frac{A}{\rho g}$	$\frac{1}{k}$

$$\Rightarrow \tau \frac{d(\Delta T)}{dt} + \Delta T = \Delta T_F$$

Taking laplace Transform on both sides

$$\Rightarrow \tau\left[S\Delta\overline{T}(S) - \Delta T(0-)\right] + \Delta\overline{T}(S) = \Delta\overline{T}_F(S)$$

$$\Rightarrow \tau S\Delta\overline{T}(S) + \Delta\overline{T}(S) = \Delta\overline{T}_F(S)$$

$$\Rightarrow \Delta\overline{T}(S)[\tau S + 1] = \Delta\overline{T}_F(S)$$

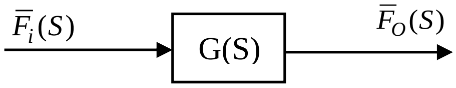
$$\frac{\Delta\overline{T}(S)}{\Delta\overline{T}_F(S)} = \frac{1}{\tau S + 1}$$

$G(S) = Transfer\ Function$

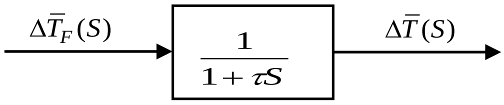
$$G(S) = \frac{Laplace\ Transform\ of\ change\ in\ output\ temp.}{Laplace\ Transform\ of\ change\ in\ input\ temp.}$$

$$G(S) = \frac{\overline{F}_O(S)}{\overline{F}_i(S)}$$

$$\Rightarrow G(S) = \frac{\Delta\overline{T}(S)}{\Delta\overline{T}_F(S)} = \frac{1}{\tau S + 1}$$



[General transfer function block]



[1st order transfer function block]

Static Characteristics is given by the equation

$$O = KI + a + N(I) + K_M I_M I + K_I I_I$$

I_0 ➔ Steady state input

$$\left.\frac{\partial O}{\partial I}\right|_{I_0} = K + K_M I_M + \frac{\partial N(I)}{\partial I}$$

Module-II

Sensing Elements

1. *Resistive sensing element*

- Potentiometer
- Thermistor
- Helipot
- Resistance Thermometer or, RTD (RTD- Resistance temp. Detector)
- Strain Gauge

It can sense temperature, strain, displacement, heat, loss etc.

$$R = \rho \frac{L}{A}$$

Potentiometer [POT]

Passive transducer: Those transducer which requires external power for energy conversion is known as passive transducer.

Active transducer: Those transducer which don't require external power sources for energy conversion, known as active transducer.

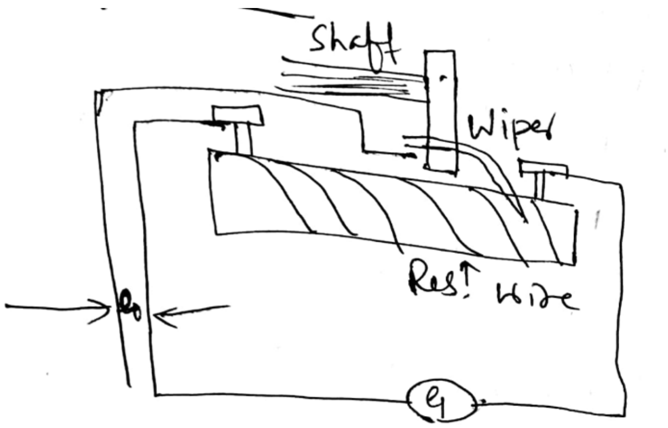
Ex: - Piezo-electric Crystal, Thermocouple

Potentiometer is used for displacement measurement.

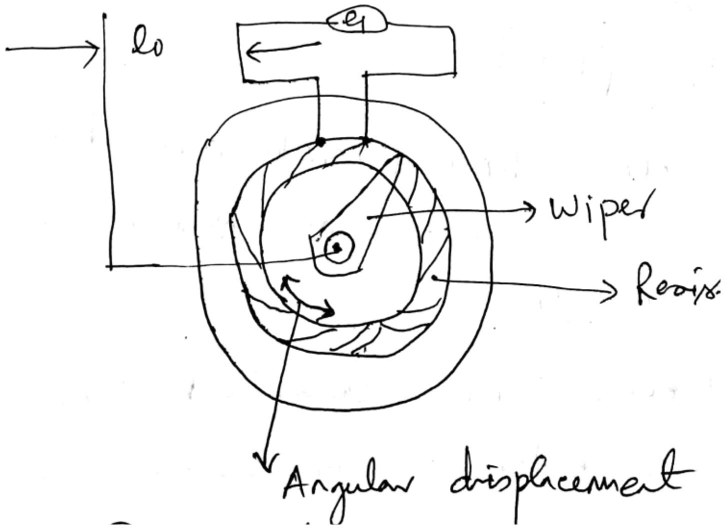
- Strain→ Strain Gauge
- Temperature→ Thermistors

It is a potential divider called POT and it is a passive sensing element.

It measures linear as well as angular displacement.

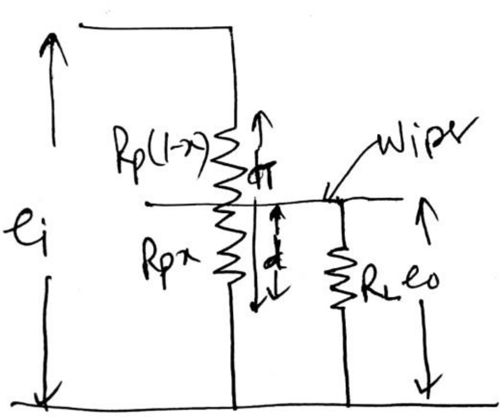


(1) Translational POT



(2) Rotational POT

Schematic Diagram:



d→length covered by the wiper

dT→Total length of the resistive element

R_P→Total Resistance of the POT

d/dt=*x*=Fractional Displacement

$$E_{th} = \frac{e_i \times R_p x}{R_p}$$

$$E_{th} = e_i . x$$

$$R_{Th} = R_p(1-x) \parallel R_p x = \frac{R_p(1-x). R_p x}{R_p} = R_p .x.(1-x)$$

$$V_L = \frac{E_{th} R_L}{R_{th} + R_L} = \frac{e_i x R_L}{R_p x(1-x) + R_L}$$

$$V_L = \frac{e_i x R_L}{R_p x(1-x) + R_L} \rightarrow Non - Linear\ Equation$$

when R_L *is* large *linearity can be obtained*

$$V_L = \frac{e_i x R_L}{R_L\left[\frac{R_p}{R_L}x(1-x)+R_L\right]} \Rightarrow V_L = \frac{e_i x}{\frac{R_p}{R_L}x(1-x)+1}$$

$$When\ R_L = \infty$$

$$V_L = e_i x \rightarrow Linear\ property\ of\ potentiometer$$

→ POT are of two types: -

i) *Wire Wound type*: - In this type 0.01 mm diameter of platinum or nickel alloy each wounded over an insulated former.

ii) *Plastic thin film type*: - These have zero resolution but have higher temperature co-efficient of resistance. This covers displacement span from 25-250mm with Non-linearity up to ±0.04% and resistance value from 500 ohm- 80 K ohm.

Resistance Thermometer (RTD)

It is made up of metal like Nickel, Cr and Pt.

→Platinum (Pt) is suitable metal for construction of RTD's because it is chemically inert and the range of temperature it can withstand is very large.

→Characteristics of Pt. with respect to temperature is linear. General relationship between change in resistance and change in temperature of any metal is given by

$$R_T = R_0 [1 + \alpha T + \beta T^2 + \Upsilon T^3 + \ldots\ldots]$$

R_T= Resistance at T ºC

R_0= Resistance at 0 ºC

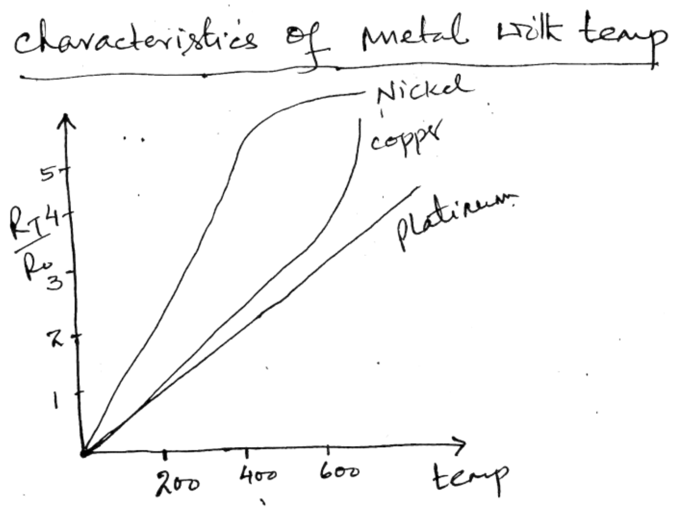
α, β, ϒ = Temperature co-efficient of resistance of the metal

→ β, ϒ are Non-linear terms and values are very small.

$$R_T = R_0 [1 + \alpha T]$$

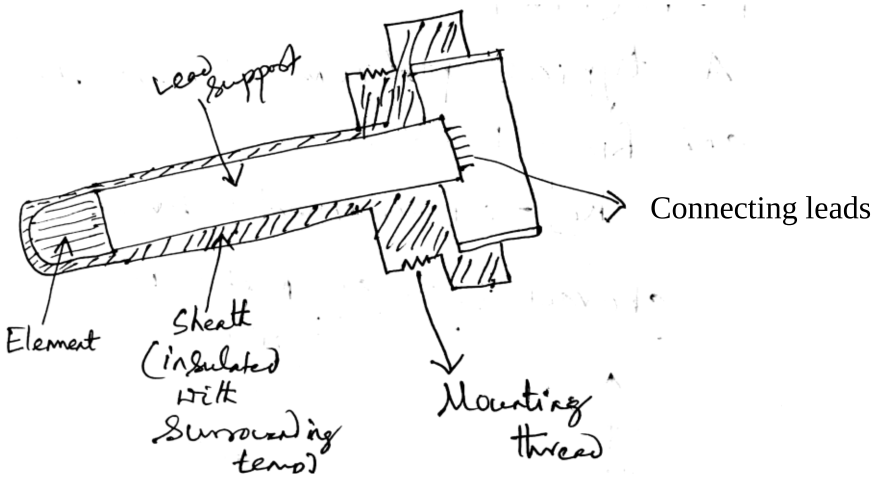
→A typical Platinum element has R_0=100 ohm and R_{100}=138.5 ohm

→[R_{100}-R_0]= Fundamental Interval



Requirement of Conductivity material to be used in RTD:

1) A change in resistance of material for unit change in temp should be as large as possible (sensitivity is High)
2) Resistance of material should have continuous and stable relationship with temperature.

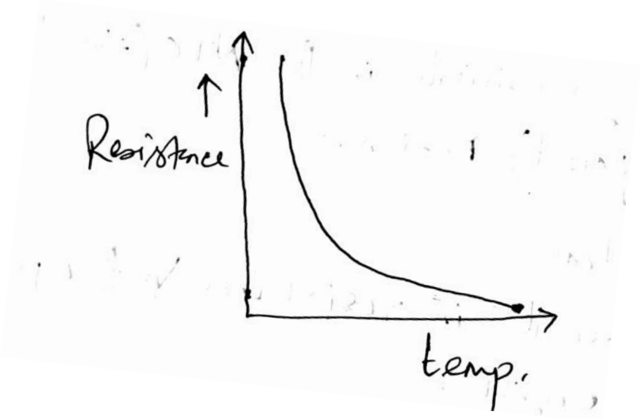


Thermistor (Thermal Resistor)

→These are resistive sensing element made up of semiconductor material used for measurement of temperature.

→The material used are metallic oxide of Cu, Ni, Fe, Co, Cr, Mn etc.

→They have very high negative (-ve) temperature co-efficient of resistance as large as several percent per degree Celsius, resistance of thermistor at room temperature may decrease as much as 5% per degree rise in temperature.



→Resistance-Temperature characteristics is given by the equation:

$$R_\theta = R_{\theta_1} e^{\beta\left[\frac{1}{\theta} - \frac{1}{\theta_1}\right]}$$

Where, Θ= Temperature to be measured in Kelvin

Θ_1= Reference Temperature

R_Θ=Resistance of temperature after rise from Θ_1 to Θ

$R_{\Theta 1}$=Resistance at reference temperature

β= Constant depends on material used for thermistor and

Its value is (3500-4500) k

Advantages:

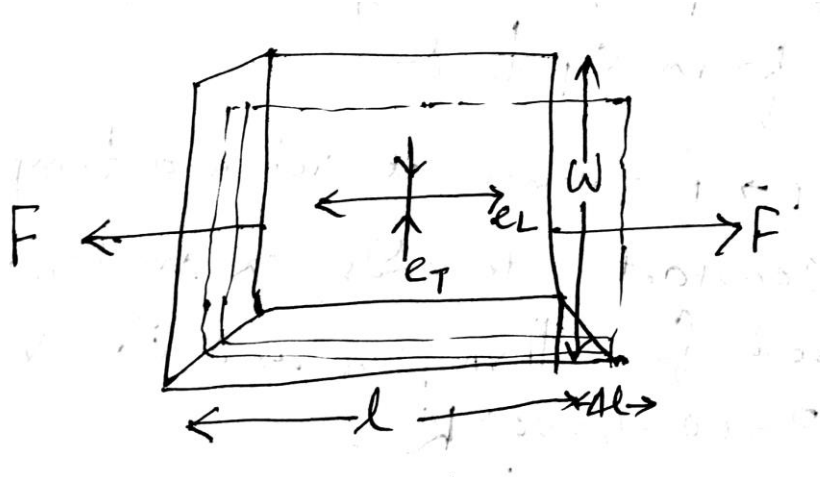
1) Small size and low cost
2) Fast Response over Narrow temperature range
3) Good Sensitivity in the NTC (-ve temp. Co-efficient) region.

Disadvantages:

1) Non-linearity in resistance vs. temperature characteristics.
2) Unstable for wide temperature range.

Strain Gauge (Piezo-Resistive Gauge)

→Strain Gauge is a metal of semiconducting element whose resistance changes when it is under strain.



Longitudinal Strain= $\frac{(l+\Delta l)-l}{l}=\frac{\Delta l}{l}=e_L$

Transverse Strain= $\frac{(W+\Delta W)-W}{W}=\frac{-\Delta W}{W}=-e_T$

Elastic Modulus= $\frac{Stress}{Strain}$

→When it is linear then it is called Young's Modulus.

Poisson Ratio:

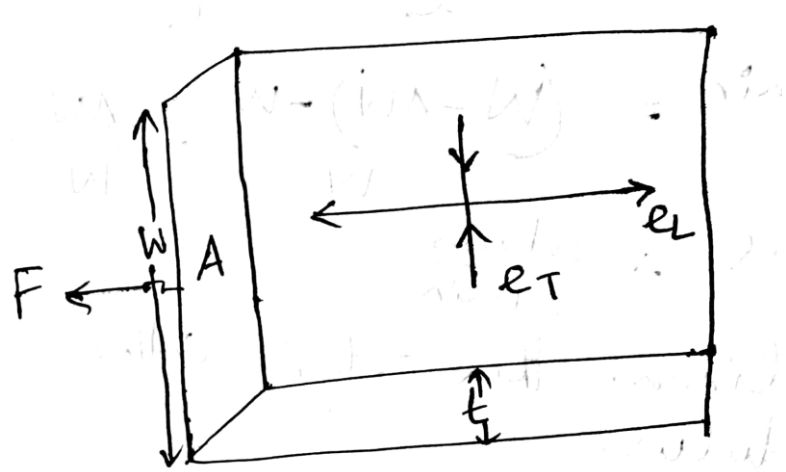
→It is the ratio of transverse Strain by Longitudinal Strain

$$\nu=-{e_T}/{e_L} \Rightarrow e_T=-\nu e_L$$

Piezo-resistive Effect:

→The change in resistivity of material when it is under strain is called Piezo resistive effect.

Derivation of Gauge Factor:



$$R = \rho \frac{L}{A}$$

$$\Delta R = \left(\frac{\partial R}{\partial L}\right)\Delta L + \left(\frac{\partial R}{\partial A}\right)\Delta A + \left(\frac{\partial R}{\partial \rho}\right)\Delta \rho$$

$$= \frac{\rho}{A}\Delta L + \frac{\rho L}{-A^2}\Delta A + \frac{L}{A}\Delta \rho$$

$$A = w \times t$$

$$\Delta A = \frac{\partial A}{\partial w}.\Delta w + \frac{\partial A}{\partial t}.\Delta t$$

$$= t\,.\Delta w + w.\Delta t$$

$$\frac{\Delta A}{A} = \frac{\Delta w}{w} + \frac{\Delta t}{t} = e_T + e_T$$

$$\Rightarrow \frac{\Delta A}{A} = 2e_T$$

$$\Delta R = \rho \frac{\Delta L}{A} - \frac{\rho L}{A} 2e_T + \frac{L}{A}\Delta \rho$$

$$\frac{\Delta R}{R} = \frac{\Delta L}{A} - 2e_T + \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta R}{R} = e_L - 2(-\nu e_L) + \frac{\Delta \rho}{\rho} = e_L + 2\nu e_L + \frac{\Delta \rho}{\rho}$$

$$\Rightarrow \frac{\Delta R / R}{e_L} = 1 + 2\nu + \frac{\Delta\rho}{e_L \rho}$$

Gauge Factor is the change in unit resistance per strain

$$GF = \frac{\Delta R / R}{e} = \frac{\Delta R / R}{\Delta L / L}$$

$$\boxed{GF = 1 + 2\nu + \frac{1}{e_L} \Delta\rho / \rho}$$

Due to change in length (1)

Due to change in Area (2ν)

Due to change in Piezo-resistive effect (very small) ($\frac{1}{e_L} \Delta\rho / \rho$)

Only for semiconductor gauge we have to consider this effect

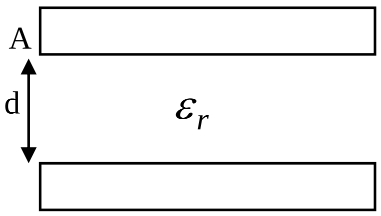
→For most of the metals Poisson ratio is approximately equals to 0.3 and the Piezo-resistive effect is 0.4.

→So, metal gauge factor is around 2.

→For semiconductor strain gauge Piezo-resistive effect term is very large. So gauge factor (GF) of Semiconductor strain gauge is very large. Hence Sensitivity is high for semiconductor type strain gauge.

→Disadvantages of semiconductor strain gauge is its resistance decreases with increase in temperature and consequently there is a decrease in gauge factor of strain gauge i.e. decrease in sensitivity.

2. *Capacitive Sensing Elements*

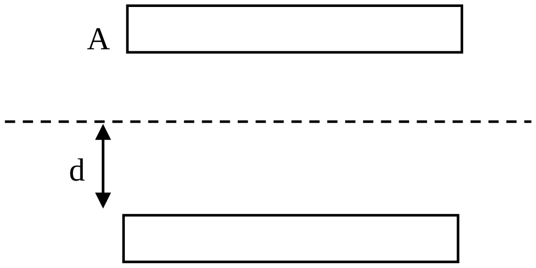


$$e = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

ε_0 =Permittivity of free space ($8.85 \times 10^{-12}\, F/m$)

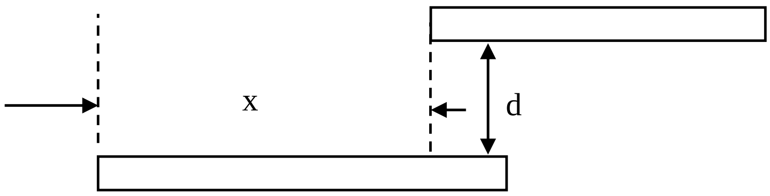
ε_r =Permittivity of relative medium

a) Variable displacement Sensor



$$e = \frac{\varepsilon_0 \varepsilon_r A}{d + x}$$

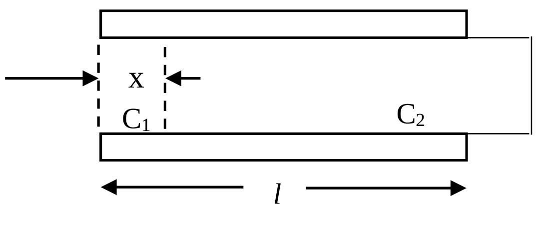
b) Variable Area Sensor



W=width of the plate

$$e = \frac{\varepsilon_0 \varepsilon_r (A - wx)}{d}$$

c) Variable Di-electric type sensor

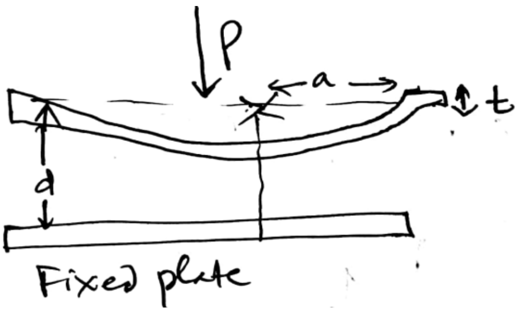


$$C = C_1 + C_2$$

$$C = \frac{\varepsilon_0 \varepsilon_{r_1} wx}{d} + \frac{\varepsilon_0 \varepsilon_{r_2} w(l - x)}{d}$$

$$\Rightarrow C = \frac{\varepsilon_0 w}{d}\left[\varepsilon_{r_2} l - x(\varepsilon_{r_2} - \varepsilon_{r1})\right]$$

d) Capacitive Pressure Sensor



$$y = \frac{3}{16} \cdot \frac{(1 - \nu^2)}{Et^3}\left(a^2 - r^2\right)\rho$$

$$\frac{\Delta C}{C} = \frac{(1 - \nu^2)a^4}{16Edt^3}$$

Where, P→Applied Pressure

ΔC →Change in Capacitance

a→Radius of the Diaphragm

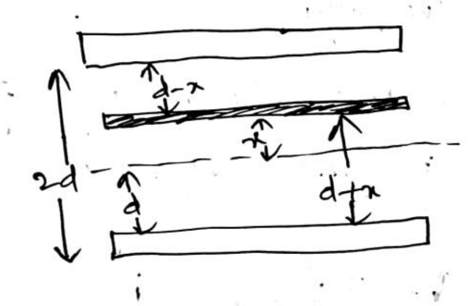
r→Radius in which we are applying pressure to generate displacement

t→Thickness of Diaphragm

E→Youngs modulus of Material

ν →Poisson's Ratio

C→Original Capacitance of capacitor before application of pressure *e) Three Plate Differential or, Push Pull Displacement Sensor*

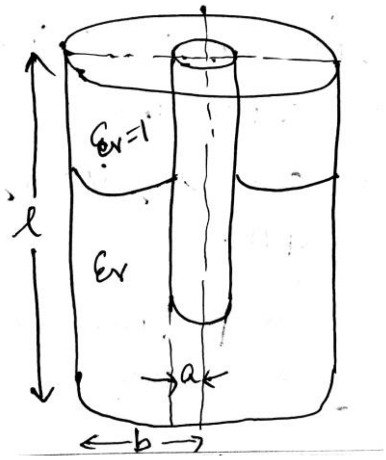


$$C_1 = \frac{\varepsilon_0 \varepsilon_r A}{d + x}$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d - x}$$

For avoiding non-linearity we have to incorporate this with a bridge circuit.

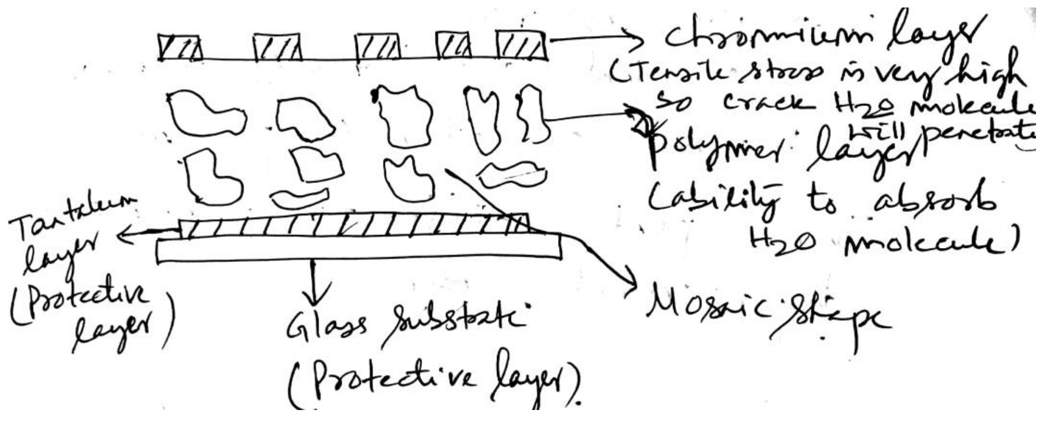
f) Capacitive level Sensor



$$\text{Capacitance / length} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log_e\left(b/a\right)}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r h}{\log_e\left(b/a\right)} + \frac{2\pi\varepsilon_0\varepsilon_r (l-h)}{\log_e\left(b/a\right)}$$

g) Thin Film Capacitance Humidity Sensor



→If all the H_20 molecule is not going to be absorbed by the polymer layer, then it won't go down.

$\Delta C \propto$ Relative Humidity

C = (375+1.7 RH) pF

RH→Relative Humidity

pF→Pico Farad

→The humidity sensor has an input range of 0 to 100% RH and a capacitance of 370 pico-Farad which RH is 0% and a linear humidity of 1.7pF per percent RH.

→The di-electric medium is characterised by a word called **loss tangent**.

$$\tan\delta = \frac{1}{\omega CR}, Q = R\sqrt{\frac{L}{C}}$$

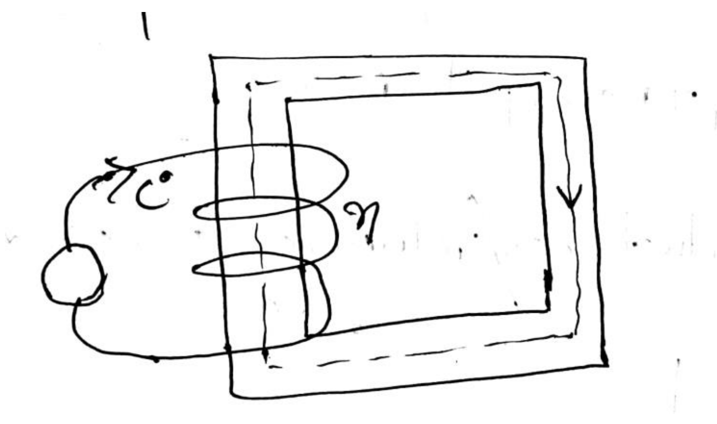
3. Inductive Sensing Element

Inductive sensing element

Variable Inductance/reluctance

Displacement Sensor

LVDT



$$mmf = \phi \times R \quad (\text{R} \rightarrow \text{Reluctance})$$

$$\phi = \frac{ni}{R}, \; N = \frac{n^2 i}{R}$$

$$L = \frac{n^2 i}{R_i} = \frac{n^2}{R}, \quad \text{Inductance(L)} = \frac{Total\ flux}{Current}$$

$$R = \frac{l}{\mu\mu_0 A}$$

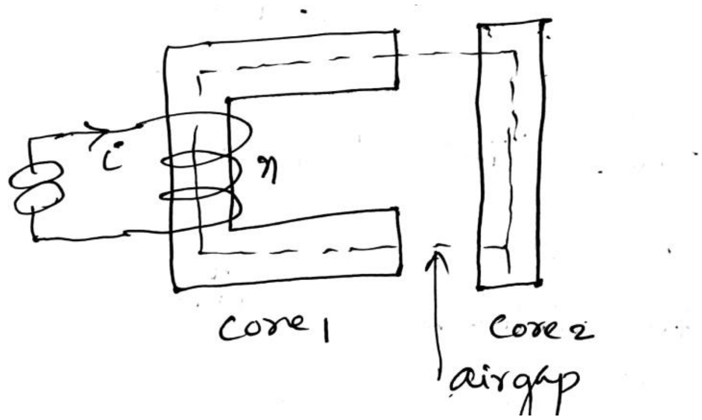
Where, l= Total length of flux path

A=Cross-sectional area of flux path

μ_0 =Permeability of Free space

$\mu_0 = 4\pi \times 10^{-7}\ Henery/m$

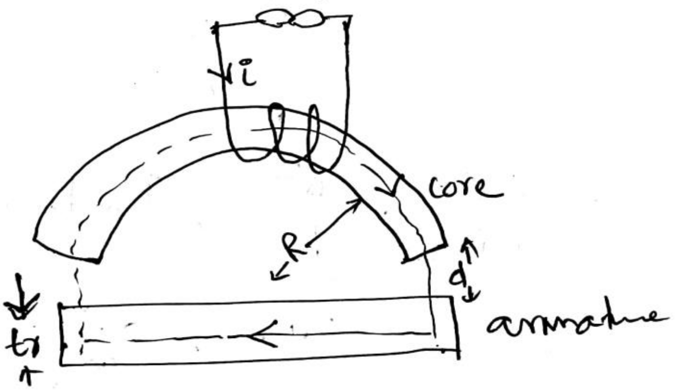
μ =Relative Permeability



$R_{total}=R_{core1}+R_{air\ gap}+R_{core2}$

Variable inductance/reluctance displacement Sensor

- Variable reluctance displacement Sensor
- Push pull or, Differential displacement sensor



$R_{total} = R_{core} + R_{air\ gap} + R_{armature}$

$$R_{core} = \frac{\pi R}{\mu_0 \mu_c \pi r^2} = \frac{R}{\mu_0 \mu_c r^2}$$

$$R_{air\ gap} = \frac{2d}{\mu_0 \pi r^2}$$

$$R_{armature} = \frac{2R}{\mu_0 \mu_A 2 r t_r} = \frac{R}{\mu_0 \mu_A r t_r}$$

$$R_{total} = \frac{R}{\mu_0 \mu_c r^2} + \frac{2d}{\mu_0 \pi r^2} + \frac{R}{\mu_0 \mu_A r t_r}$$

$$= \frac{R}{\mu_0 r}\left[\frac{1}{\mu_c r} + \frac{1}{\mu_A t_r}\right] + \frac{2d}{\mu_0 \pi r^2}$$

$$R_T = R_0 + kd$$

$$\Rightarrow R_0 = \frac{R}{\mu_0 r}\left[\frac{1}{\mu_c r} + \frac{1}{\mu_A t_r}\right]$$

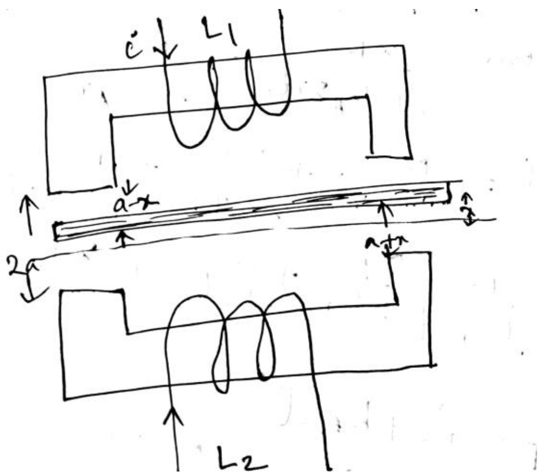
$$k = \frac{2}{\mu_0 \pi r^2}$$ [k value depends upon the structure of the material]

$$L_T = \frac{n^2}{R_T} = \frac{n^2}{R_0 + k.d} = \frac{n^2 / R_0}{R_0 / R_0 + k.d / R_0} = \frac{L_0}{1 + \alpha d}$$

$$\left[\therefore L_0 = n^2 / R_0 \ \ and \ \ \alpha = k / R_0\right]$$

L_0→Inductance of sensor at zero air gap, k & α depends upon the structure of the sensor.

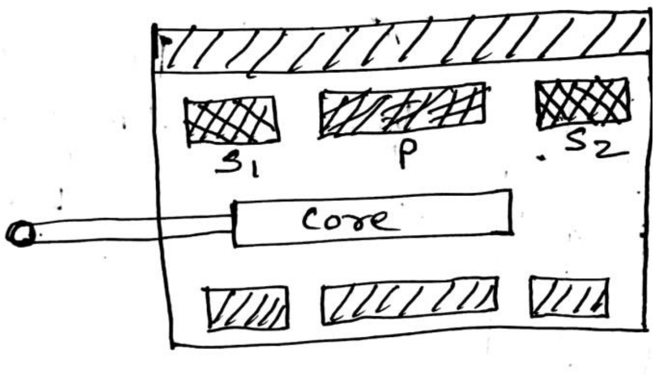
Push pull or, Differential displacement sensor



$$L_1 = \frac{L_0}{1+\alpha\,(a-x)}, \quad L_2 = \frac{L_0}{1+\alpha\,(a+x)}$$

→In order to avoid non-linearity, we design this type of sensor. We have to incorporate this with bridge circuit.

LVDT (Linear Variable Differential Transformer)



→ The soft iron core makes a flux linkage between the primary winding and the two secondary winding.

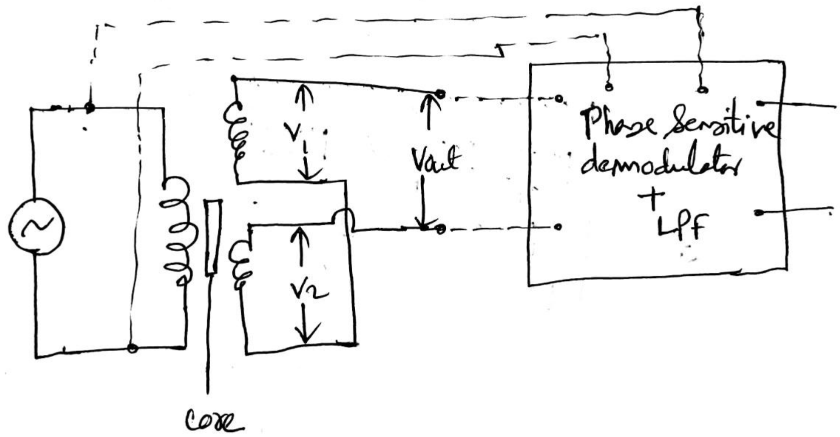
→The two secondary windings are identically placed either side of the primary windings.

→Number of turns of two secondary windings are equal.

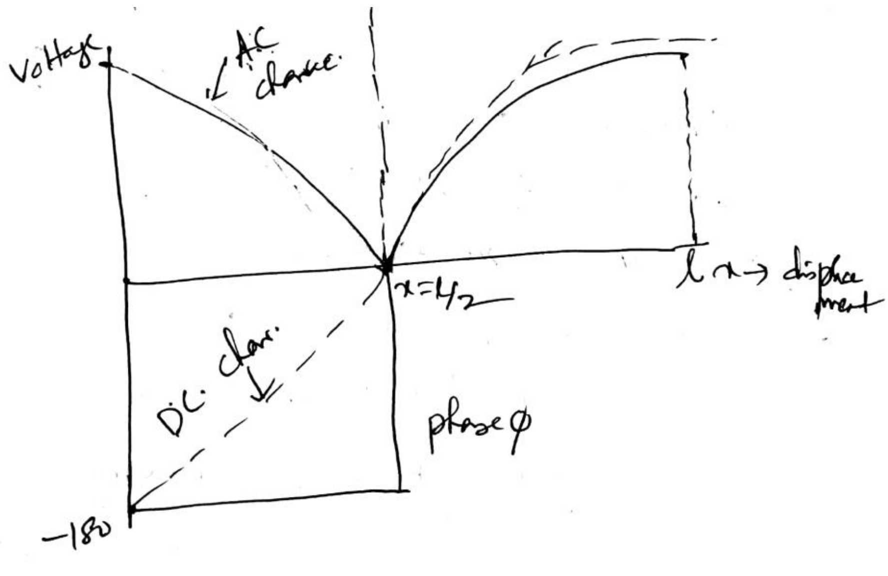
→The core is made up of high permeability (unwanted fizzer component) Nickel iron which is having low harmonics, low null voltages and is of high sensitivity.

→The core is slotted longitudinally to reduce the eddy current loss. (Current due to the back e.m.f.)

LVDT Electrical equivalent Circuit:



→The two secondary coils are connected in series so as to get one output.



→Phase sensitive demodulator- to produce positive voltage and negative voltage.

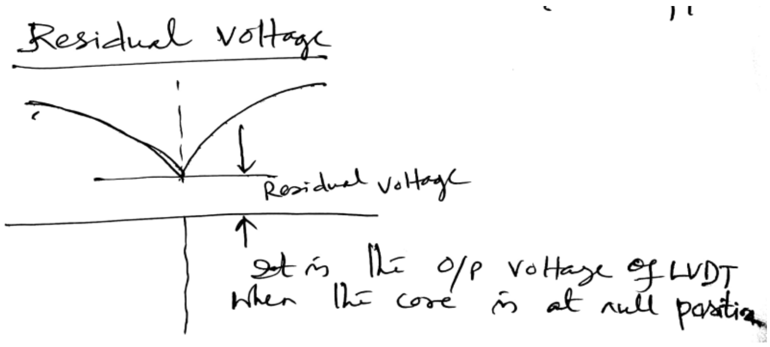
Note

Low Pass Filter (LPF)

→The Band width B.W. of LPF is less than the sampling frequency to filter out the carrier signal and more than maximum frequency not to filter out the measurement signal.

→Frequency of AC applied to primary winding is in the range of 50Hz-20KHz. The Primary winding is excited by AC source produce an alternating magnetic field which includes A.C voltage in the two secondary winding.

→To represent the output from the two secondary's in to a single voltage, the two secondaries are connected in series opposition.



Reason for Occurrence of Residual Voltage in LVDT

- This may be due to an account of presence of harmonics in the input supply voltage and also the harmonics present or produced in the output voltage.
- The Residual voltage may be due to either an incomplete magnetic or, electrical unbalance.

4. *Electromagnetic Sensing Element*

→Eg: - Velocity Sensor

Variable Reluctance Tacho generator for measurement of angular velocity

→These sensors are based on Faraday's laws for measurement of linear and angular velocity.

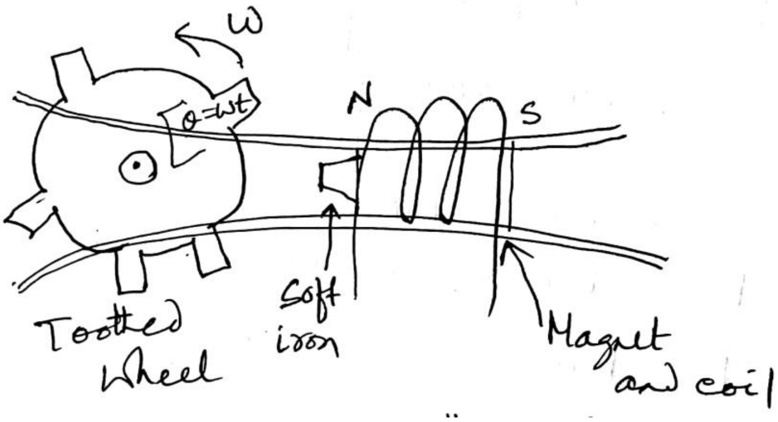
→The induced emf in a conductor depends on the rate of change of flux linking with conductor.

$$E = -\frac{dN}{dt} \quad N \rightarrow Total \ \ flux$$

Construction:

→It consists of a toothed wheel of ferro-magnetic material and a coil wounded on a permanent magnet extended by a soft iron pole piece.

→When the tooth is close to the pole piece reluctance is minimum but the reluctance increment with the tooth moves away from the pole piece.



→Reluctance is maximum when the gap is adjustant to the pole piece and falls again as the next tooth approaches to pole piece.

The total flux linked by a core of 'n' turn coil is

$$N(\theta) = a + b\cos m\theta$$

Where, a= mean flux

b= amplitude

m= no. of teeth

$$E = -\frac{dN}{dt} = -\frac{dN}{d\theta} \times \frac{d\theta}{dt}$$

$$= bm\sin(m\theta)\frac{d\theta}{dt}$$

$$= bm\sin(m\theta)\omega_r$$

$$E = bm\omega_r \sin(m\omega t) \quad [\because \theta = \omega t]$$

$$\hat{E} = bm\omega_r \quad f = \frac{m\omega}{2\pi}$$

5. *Thermoelectric Sensing element (Thermocouple)*

Seeback effect: - It states that when two different metal of different composition are joined to form two junction and if the temperature of the junction is changed then potential difference will be developed across the junction and this potential difference is called Seeback emf.

$$E_T^{AB} = a_1T_1 + a_2T_2^2 + a_3T_3^3 + ...$$

Where, a_1, a_2, a_3 →Constants which depends on the type of metals

→*Thermocouple:* - A thermo couple is constructed when two dissimilar metal of different composition are joined to form a close circuit consisting of two junction J_1 and J_2 and the two junctions are maintained at different temp. T_1 and T_2 respectively.

→*Hot Junction/ Measuring Junction:* - This junction is inserted into the system for measurement of the process temperature.

→*Cold Junction/ Reference Junction:* -This Junction is usually kept at reference temperature of 0 ºC for most industrial applications.

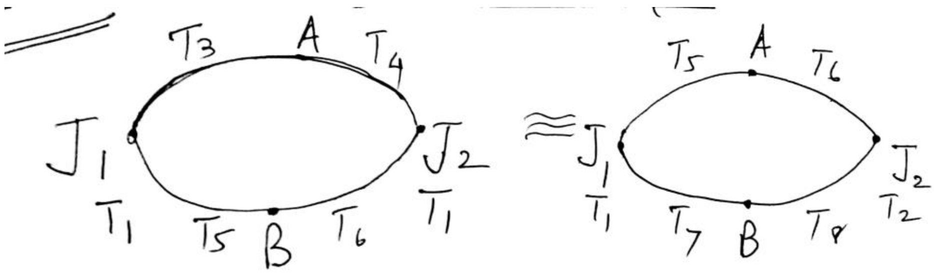
$$E_{T_1T_2}^{AB} = a_1(T_1 - T_2) + a_2(T_1^2 - T_2^2) + a_3(T_1^3 - T_2^3) + ...$$

T_1= Temperature of Measuring Junction.

T_2= Temperature of Reference Junction.

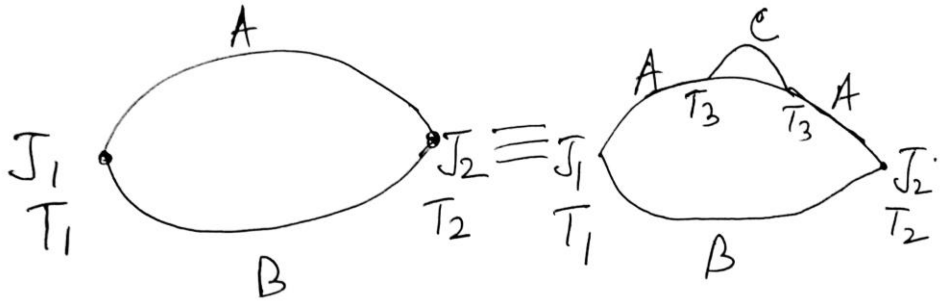
Laws of Thermocouple: -

Law-I



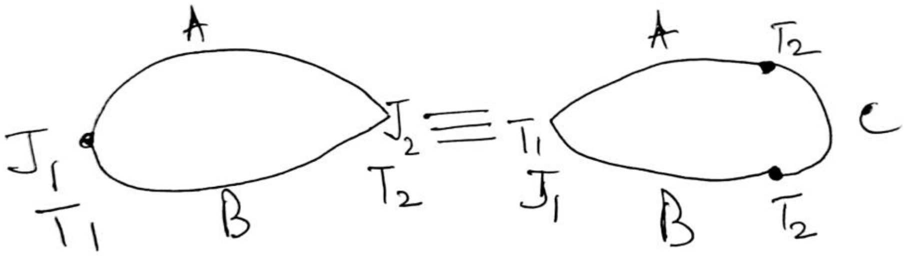
→It states that the emf of a given thermocouple depends only on the temperature of the junction and independent of the wire connecting the junction.

Law-II



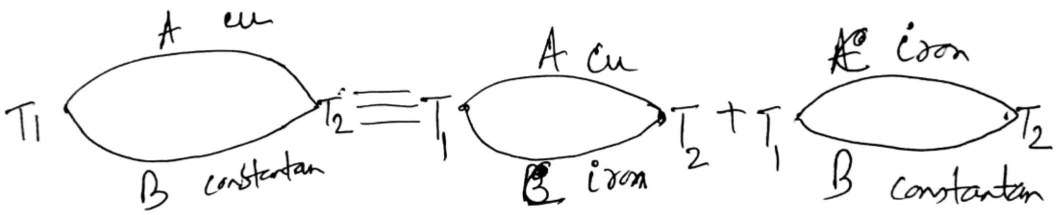
→This law states that if a 3rd metal is introduced into A (or B) and the two new junctions are at same temperature T_3, then emf of the thermocouple is unchanged.

Law-III



→This law states that if a 3rd metal 'C' is inserted between A and B at either junction. And the new junction AC or BC at the same temperature (T_1 or T_2), then the emf is unchanged.

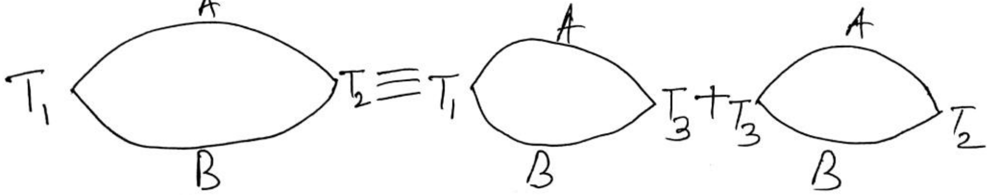
Law-IV



→Law of intermediate metal

$$E_{T_1 T_2}^{AB} = E_{T_1 T_2}^{AC} + E_{T_1 T_2}^{CB}$$

Law-V

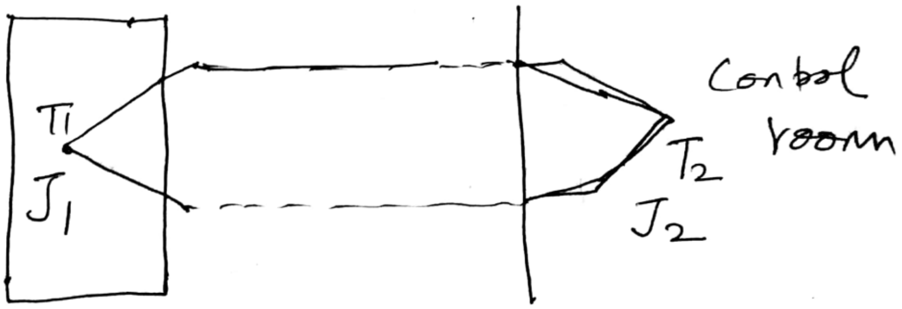


→Law of intermediate Temperature

$$E_{T_1T_2}^{AB} = E_{T_1T_2}^{AC} + E_{T_1T_2}^{CB}$$

Lead Compensation

→The use of extra lead in a thermocouple for compensating the changing output voltage of the thermocouple due to temperature variation is called lead compensation.

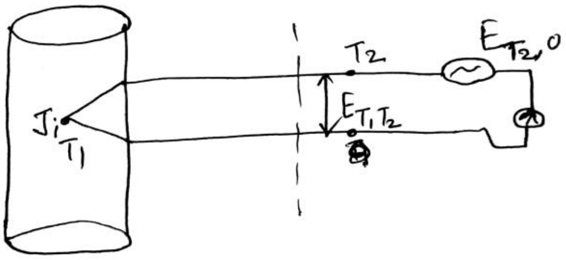


→The extra leads used for lead compensation may be of same material of thermocouple or, of different material having the same thermal conducting properties of the metal used.

Installation Problem

- Temperature indication too low with a very thin thermocouple.
- Varying temperature indication with proper operation.
- Temperature indication error increases with increasing temperature (low indication).
- Large deviations of the temperature indication from the default values.
- Temperature indication changes over the course of time.
- Temperature indicating instrument shows room temperature.
- Negative temperature indication.
- Temperature indication error in the range of 20-25 °C
- Temperature indication even when thermocouple is disconnected.

Reference Junction Compensation of Thermocouple: -



Case-I

$T_1=100, T_2=0, a_1=4$

$E=a_1 (T_1- T_2) =4(100-0) =400mV$

Case-II

$T_1=100, T_2=1, a_1=4$

$E=a_1 (T_1- T_2) =4*99=396mV$

So error = 4mV

→Due to change in atmospheric temperature the reference junction temperature T_2 will be changed and there will be a corresponding change in output voltage of the thermocouple. To compensate this change in output voltage with atmospheric temperature variation is to be compensated by introducing a 2nd source of emf $E_{T_2,0}$ in series with the thermocouple output voltage (E_{T_1,T_2}).

$$E_{T_1,0} = E_{T_1,T_2} + E_{T_2,0}$$

→Metal resistance thermometer is incorporated to bridge circuit will sense the change in temperature at the reference junction and giving an output voltage signal proportional to the reference junction temperature. So output of bridge circuit should be equal to zero.

→Output of bridge circuit incorporating thermometer or, RTD is given by:

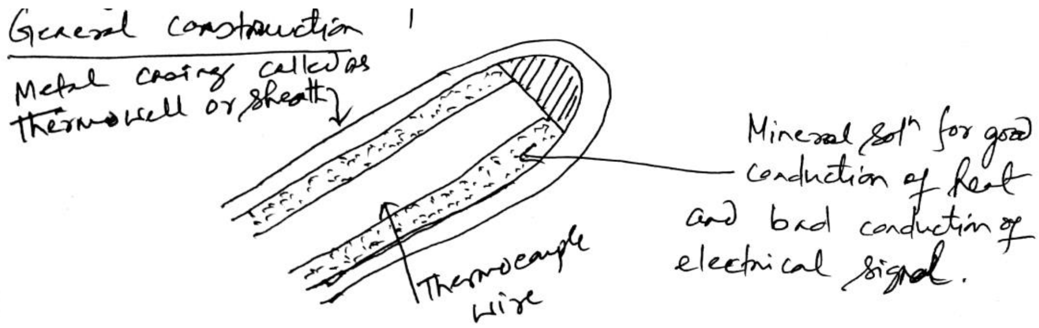
$$V_{out} = V_s \frac{R_2}{R_3} \alpha T_2$$

Where, α= resistance temperature co-efficient of RTD

Peltier Effect

→This is the reverse of see-back effect when the potential difference is applied across the two junction of thermocouple, one junction of thermocouple will be heated and junction temperature will be increased and other junction temperature will be decreased.

General Construction: -



IC Temperature Sensor

→Small size.

→No compensation circuit is required i.e. it is already present inside it.

→ These IC generate electrical output proportional to the temperature. The sensor works on the principle that the forward voltage of a silicon diode depends on its temperature.

→Voltage Temp. Characteristics:

$$V_F = \frac{KT}{e} \ln \frac{I_F}{I_S}$$

Where, T=Ambient temp in K

K=Boltzmann Constant =1.3867*10^{-23} J/k

E= Charge of electron

I_F=Forward Current

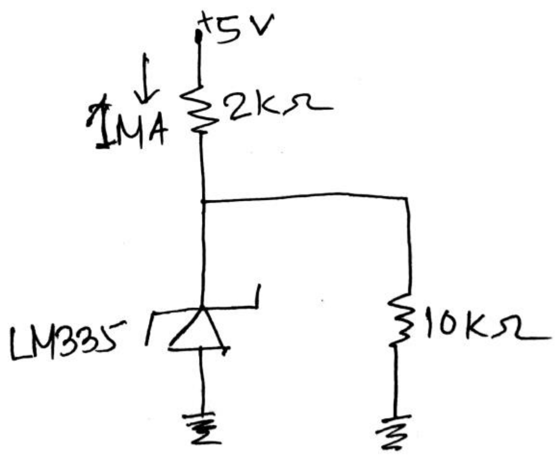
I_S=Saturation Current

→Two diodes are used to avoid saturation current.

→IC temperature sensors are available in both voltage and current output configuration.

→The current output units are usually set for an output change 1μA per Kelvin while the voltage output configuration generates 10mV per K.

LM 335



→It operates as a two terminal Zener diode with an output voltage of 10mV/K.

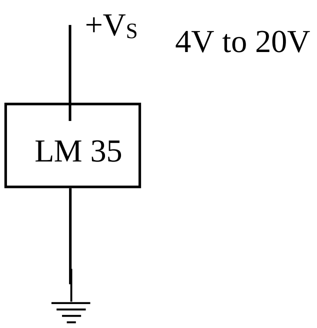
→Temperature Range is -55 ºC to 125 ºC.

→It gives linear output.

→Current range 400 μA< I_Z < 5mA [I_Z→Zener diode].

Device Name	Temp. range	Use
LM 135	-55 ºC to 150 ºC	Defence
LM 235	-45 ºC to 125 ºC	Industrial
LM 335	-45 ºC to 100 ºC	Commercial

LM 35



→Temperature Range is -55 ºC to 150 ºC

→Its output voltage of 10mV/ ºC

→It gives a nearly linear output characteristics.

→It behaves as a three terminal reference rather than a two terminal Zener powered by a 4V to 20V applied to 3rd terminal. But +ve and –ve supply with a pull down resistance are necessary to operate in near or below 0 °C.

AD 592

→It is a 2-terminal device that acts like a constant current element passing a constant current in µA equal to absolute temperature.

→Temperature range -25 ºC to 105 ºC.

→Output Current range 1 µA/K.

6. Elastic Sensing Element

→It converts an input force into an output displacement.

→It is commonly used for measuring torque, pressure, acceleration etc.

Burdon tube

→It is used for pressure measurement.

→These are of various types

- C-type
- Spiral type
- Helical type

→The Burdon tubes are made out of an elliptically flattered tube beat in such a way as to produce the different shape.

→One end of the tube is sealed and other is open.

→When the fluid whose pressure is to be measured enters the tube there tends to straighten out an account of the pressure applied.

→This causes a movement of free end and displacement of this end is amplified through a mechanical linkage.

→The amplified displacement of the free end may be used to move a pointer on scale calibrated in terms of applied pressure.

→Burdon tubes are made up of different materials which includes brass, alloy steel, stainless steel, bronze, phosphorous bronze, beryllium copper, monel etc.

Monel→ High pressure measurement

→Phosphorous bronze is used in low pressure application where the atmosphere is non-corrosive, while in application where corrosion and/or high pressure is a problem stainless steel or, Monel are used.

Advantages: -

→Low Cost, Simple Construction.

→Wide variety of ranges, high accuracy especially in relation to cost.

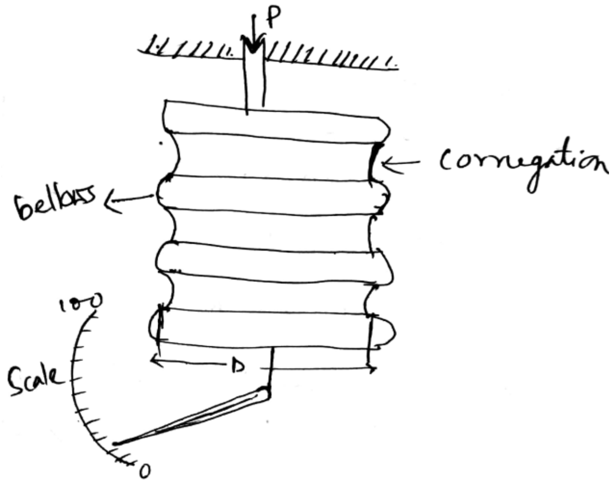
Disadvantages: -

→Low spring gradient.

→Susceptible to vibration and hysteresis.

→Susceptible to electric shock.

Bellows



→Bellow Convert pressure to displacement.

→Metallic bellows is a series of circular parts of metal welded together (Bronze or alloy)

→The bellow are made up of an alloy which is ductile, has high strength and retains its properties over long use that has very little hysteresis effect.

→Relation between the applied pressure and displacement is given by:

$$d = \frac{0.453 P b n D^2 \sqrt{1-\nu^2}}{E t^3}$$

Where, P=Pressure

b=Radius of each corrugation

n=no. of Semiconductor corrugation

t=thickness of wall

D=Mean diameter

E=Young's Modulus of Electricity

ν =Poisson's ratio

Advantages

→Cost is moderate, able to deliver high force.

→It is adaptable for absolute and differential pressure.

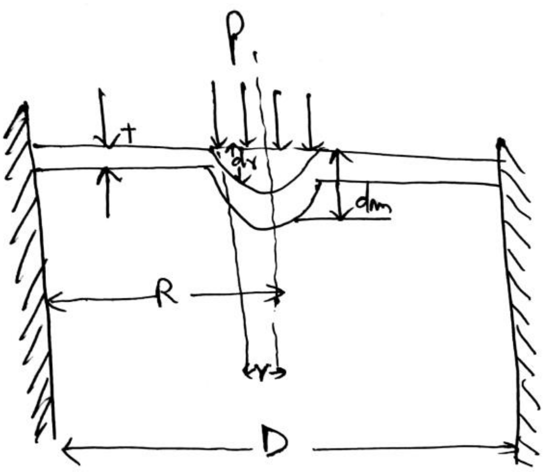
→It is good in low to moderate pressure range.

Disadvantages

→It needs ambient temperature compensation

→It is unsuitable for high Pressure.

Diaphragm



→Diaphragm is pneumatic sensor which convert pressure to displacement.

→Diaphragm is nothing but an elastic membrane on which pressure is applied and the corresponding displacement is measured.

→For this type of diaphragm, the relationship between the applied pressure and displacement is given by:

$$P = \frac{256E\,t^3 d_m}{3(1-\nu^2)D^4} \quad N/m^2$$

Where, E=Young's Modulus

t=thickness of diaphragm

d=diameter of the diaphragm

R=Radius of the diaphragm

ν =Poisson's Ratio

d_m=deflection at centre of diaphragm corrugated environment, we can also use rubber.

Practical Elastic Sensor

1) Cantilever load cell

$$e = \frac{b(l-x)F}{wt^2\varepsilon}$$

Where, e→Total force induced on applying force

➔ The applied force causes the cantilever to bend so that top surface experiences a tensile strain and the strain gauge one and there will experience an increase in resistance.

$$R_1 = R_3 = R_0 + \Delta R$$

Where, R_0 =initial resistance at t (-0)

➔ Strain gauge 2 and 4 experience compressive strain and there will be decrease in resistance $R_2 = R_4 = R_0 - \Delta R$

$$G = \frac{\left(\Delta R / R_0\right)}{e}$$

$$\Delta R = GeR_0$$

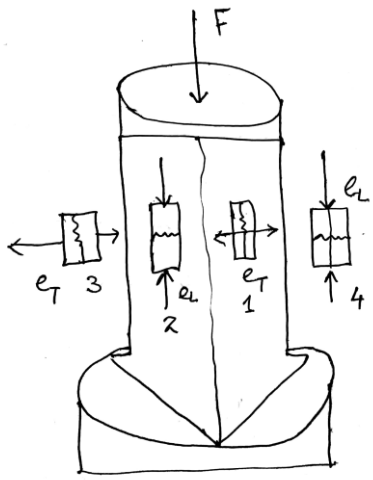
$$R_1 = R_3 = R_0 + R_0 Ge$$

$$= R_0(1+Ge)$$

$$R_2 = R_4 = R_0(1-Ge)$$

$$E_{th} = V_S Ge \rightarrow Output\ voltage\ of\ bridge\ ckt.$$

2) Pillar Load cell



$$E = \frac{Stress}{Strain} = \frac{F/A}{-e_L}$$

$$e_L = -F/AE.$$

$$e_T = -\nu e_L = \frac{F\nu}{AE}$$

$$R_1 = R_3 = R_0 + \Delta R$$

$$R_2 = R_4 = R_0 + \Delta R$$

$$\Delta R = GeR_0$$

$$R_1 = R_3 = R_0 + R_0 Ge_T$$

$$= R_0(1+Ge_T)$$

$$R_2 = R_4 = R_0 + R_0 Ge_L$$

$$= R_0(1+Ge_L)$$

$$R_1 = R_3 = R_0\left(1 + \frac{GF\nu}{AE}\right)$$

$$R_2 = R_4 = R_0\left(1 - \frac{GF}{AE}\right)$$

$$E_{th} = \frac{V_S}{2}\left(\frac{GF}{AE}\right)(1+\nu) \rightarrow Output\ of\ the\ bridge\ ckt$$

Torque Sensor

→Torque Sensor is a 2nd order dynamic element.

$$Total\ e = \frac{T}{\pi S a^3}$$

Where, S= shear Modulus

T= Applied Torque

a=Radius of cylinder

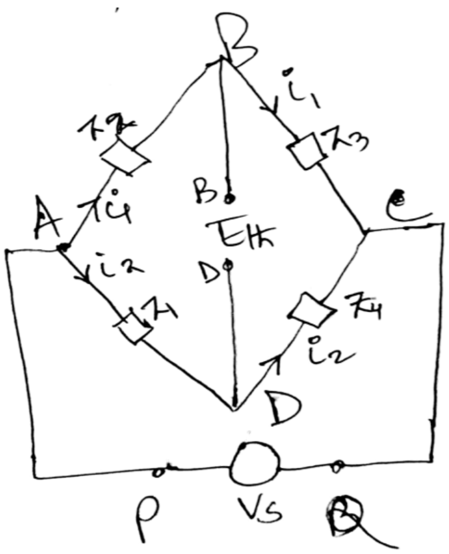
$$R_1 = R_3 = R_0(1 + Ge)$$

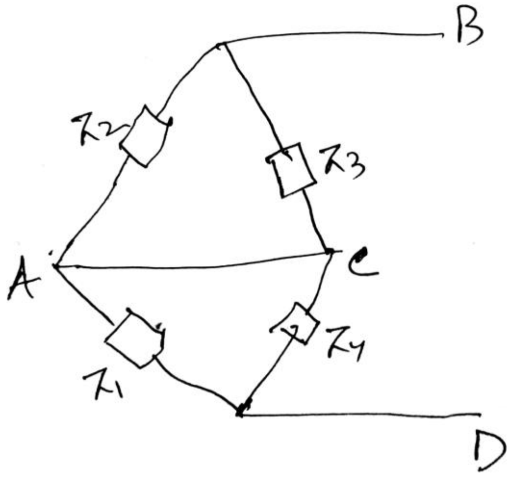
$$R_2 = R_4 = R_0(1 - Ge)$$

$$E_{th} = V_S Ge$$

Module-III

→Signal Conditioning element measure and detect the output of resistive, capacitive and inductive sensor.





$$Z_{th} = \frac{Z_2 Z_3}{Z_2 + Z_3} + \frac{Z_1 Z_4}{Z_1 + Z_4}$$

Taking the loop PABCQ

$$V_s - i_1(Z_2 + Z_3) = 0$$

$$\Rightarrow i_1 = \frac{V_s}{(Z_2 + Z_3)}$$

Taking the loop PADCQ

$$V_s - i_2(Z_1 + Z_4) = 0$$

$$\Rightarrow i_2 = \frac{V_s}{(Z_1 + Z_4)}$$

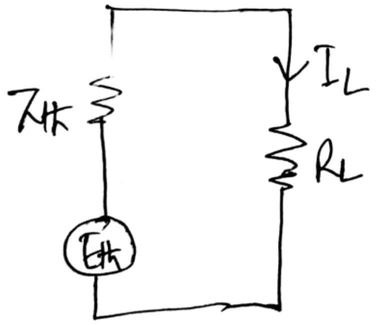
$$E_{th} = V_B - V_D$$

$$= V_S - i_1 z_2 - (V_s - i_2 Z_1)$$

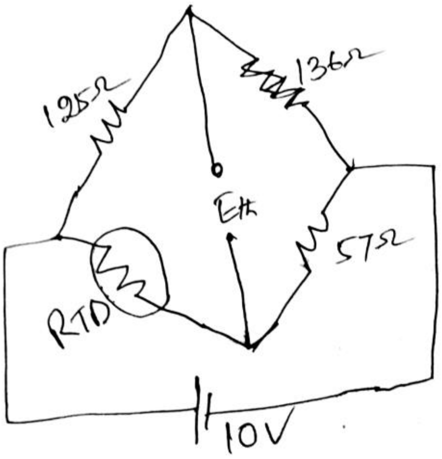
$$= V_S - i_1 z_2 - V_s + i_2 Z_1$$

$$= -\frac{V_S Z_2}{Z_2 + Z_3} + \frac{V_S Z_1}{Z_1 + Z_4}$$

$$= V_S \left[\frac{Z_1}{Z_1 + Z_4} - \frac{Z_2}{Z_2 + Z_3} \right]$$



$$V_L = E_{th} \frac{R_L}{R_L + Z_{th}}$$



Q. If the RTD is working on temperature 100 ºC and R_0 =100 ohm then find out E_{th} & Z_{th}? α=0.1

$R_T = R_0 (1+\alpha T)$

=100. (1+10) = 1100 ohm

$E_{th} = 10\left[\frac{1100}{1157} - \frac{125}{125+136}\right]$ = 4.718 V

$Z_{th} = \frac{125 \times 136}{125+136} + \frac{1100 \times 57}{1100+57}$ =119.325 ohm [Note: $Z_L >> Z_{th}$]

Notes

→The null type Wheatstone bridge is accurate but the problem with this bridge is that the balancing even if done automatically is not instantaneous. Therefore, this bridge is unsuitable for dynamic application when the changes in resistance are applied.

→For measurement of rapidly changing input signal the deflection type bridge is used.

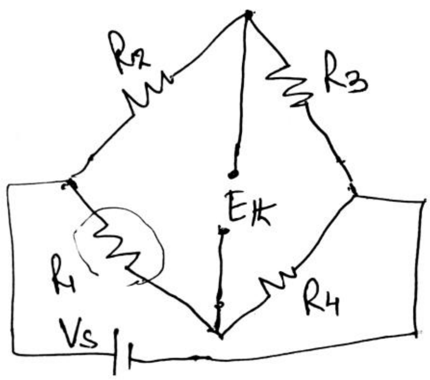
Current Sensitivity Bridge:

→If the output of the bridge is connected directly to a low impedance device like galvanometer or, PMMC instrument a large current flows through the meter. In this case, the bridge is called a current sensitivity bridge.

Voltage Sensitivity Bridge:

→In most of the application of the deflection type bridge, the bridge output is fed to an amplifier which has high input impedance and therefore the output current is zero that is connected to a CRO or, an electronic voltmeter or digital voltmeter and this is called voltage sensitive bridge.

Design of Resistive Deflection Bridge



$$E_{th} = V_S \left[\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right]$$

Replacing R_1 by temperature sensor R_I

$$E_{th} = V_S \left[\frac{R_I}{R_I + R_4} - \frac{R_2}{R_2 + R_3} \right]$$

$$E_{th} = V_S\left[\frac{1}{1+\frac{R_4}{R_I}} - \frac{1}{1+\frac{R_3}{R_2}}\right]$$

→To design a single element bridge we need to specify the 3 parameter i.e. V_S, R_4 and ratio of R_3 and R_2

When minimum case

$$V_{\max} = V_S\left[\frac{1}{1+\frac{R_4}{R_{I\max}}} - \frac{1}{1+\frac{R_3}{R_2}}\right]$$

For balancing a bridge put $V_{min} = 0$

$$V_S\left[\frac{1}{1+\frac{R_4}{R_{I\min}}} - \frac{1}{1+\frac{R_3}{R_2}}\right] = 0$$

$$\Rightarrow \frac{1}{1+\frac{R_4}{R_{I\min}}} = \frac{1}{1+\frac{R_3}{R_2}}$$

$$\Rightarrow \frac{R_4}{R_{I\min}} = \frac{R_3}{R_2} \quad \rightarrow \textit{Balanced Condition}$$

→An important consideration is needed to limit the electrical power I^2R in the sensor to a level which enables it to be dissipated as heat flow to the surroundings fluid otherwise the temperature of the sensor rise above that of the surrounding fluid thereby affecting the sensor resistance.

$$W \geq V_s^2 \left[\frac{R_1}{(R_1 + R_4)^2} \right]$$

$$W \Rightarrow Power$$

→Another requirement is the need to keep the non-linearity of the overall relationship between E_{th} and current (I) with specified unit.

If $V_{min} = 0$ and V and I makes a straight line

$$V_{ideal} - V_{min} = \frac{V_{\max} - V_{\min}}{I_{\max} - I_{\min}} (I - I_{\min})$$

$$\Rightarrow V_{ideal} = \frac{V_{\max}}{I_{\max} - I_{\min}} (I - I_{\min})$$

$$\Rightarrow V_{ideal} = \frac{V_{\max} I}{I_{\max} - I_{\min}} - \frac{V_{\max} I_{\min}}{I_{\max} - I_{\min}}$$

$$N(I) = E_{th} - V_{ideal}$$

$$\widehat{N}(I) \geq \frac{E_{th} - V_{ideal}}{V_{\max}} \times 100$$

$$E_{th} = V_S \left[\frac{1}{1 + \frac{R_4}{R_I}} - \frac{1}{1 + \frac{R_3}{R_2}} \right]$$

Under Balanced condition

$$\frac{R_4}{R_{I_{\min}}} = \frac{R_3}{R_2}$$

$$\Rightarrow R_4 = \frac{R_3}{R_2} \times R_{I_{\min}}$$

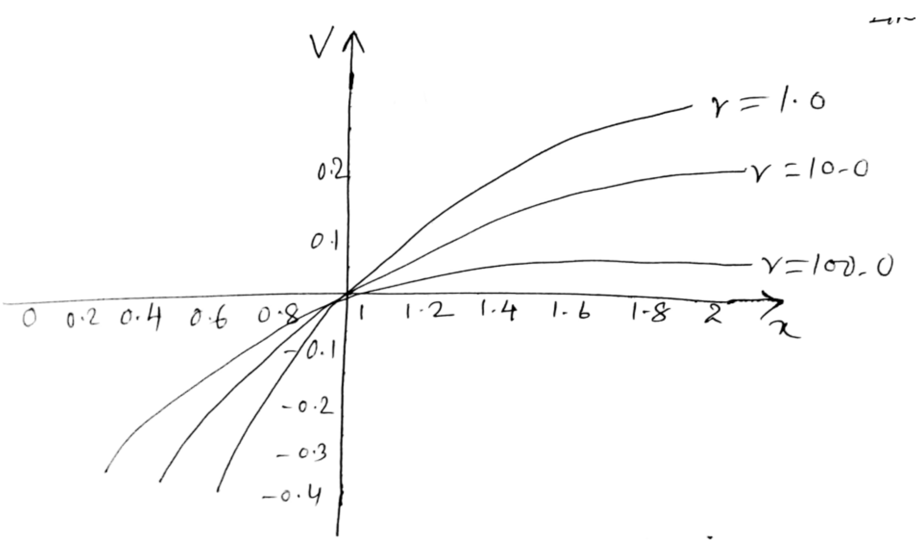
$$E_{th} = V_S\left[\frac{1}{1+\frac{R_3}{R_2}\times\left(\frac{R_{I\min}}{R_I}\right)} - \frac{1}{1+\frac{R_3}{R_2}}\right]$$

$$E_{th} = \left[\frac{1}{1+\frac{R_3}{R_2}\times\left(\frac{R_{I\min}}{R_I}\right)} - \frac{1}{1+\frac{R_3}{R_2}}\right]$$

$$V = \frac{1}{1+\frac{r}{x}} - \frac{1}{1+r}$$

$$\Rightarrow V = \frac{x}{x+r} - \frac{1}{1+r}$$

$$V = \frac{E_{th}}{V_S}, \quad r = \frac{R_3}{R_2}, \quad x = \frac{R_I}{R_{I\min}}$$



→From the graph it is found that V is zero at x =1 corresponding to the bridge being balanced at I = I_{min}.

Case-I

$$x = \frac{R_I}{R_{I_{\min}}} = 1$$

V = 0

$E_{th} = 0$

This is the required balanced condition of the bridge circuit.

→A bridge is balanced when $R_I = R_{I\min}$. If we consider the resistive sensor the deflection bridge is a strain gauge then, $R_I = \Delta R$

$$R_{I_{\min}} = 0$$

$$\frac{\Delta R}{R} = Ge$$

Where, G= Gauge Factor

e = Strain

→Maximum sensitivity of the bridge is achieved for x = 1

$$\left.\frac{\partial V}{\partial x}\right|_{x=1} \text{ where } r = 1$$

$$\frac{R_3}{R_2} = 1\ ,\ R_4 = \frac{R_3}{R_2} \times R_{I_{\min}}$$

$$R_4 = R_{I_{\min}}\ ,\ R_4 = R_0 = \text{Constant resistance}$$

→Thus for a bridge with a single strain gauge we require R_3, R_4, R_2, and all should be equal to R_0 i.e. the unstrained resistance.

$$V = \frac{x}{x+r} - \frac{1}{1+r}\ \ , put\ r = 1$$

$$\Rightarrow V = \frac{x}{x+r} - \frac{1}{2} = \frac{2x - x - 1}{2(x+1)}$$

$$\Rightarrow V = \frac{x-1}{2(x+1)}$$

$$put\ x = \frac{R_I}{R_{I_{\min}}} \Rightarrow V = \frac{\left(\frac{R_I}{R_{I_{\min}}} - 1\right)}{2\left(\frac{R_I}{R_{I_{\min}}} + 1\right)}$$

$$\Rightarrow V = \frac{\left(R_I - R_{I_{\min}}\right)}{2\left(\frac{R_I}{R_{I_{\min}}} + 1\right).R_{I_{\min}}} = \frac{\Delta R}{2.(2)R_{I_{\min}}}$$

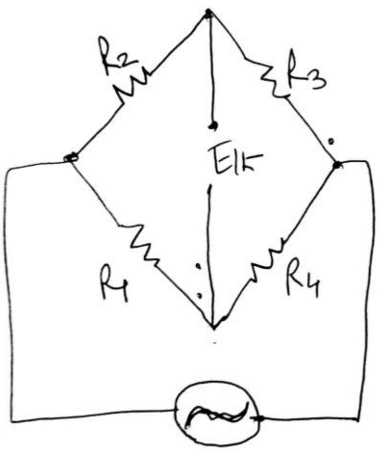
$$\because \frac{\Delta R}{R_{I_{\min}}} = G\,e$$

$$\Rightarrow V = \frac{1}{4} G\,e$$

$$E_{th} = \frac{V_s G e}{4}$$

→This is the output voltage equation of a bridge element in single strain gauge in one arm of the bridge circuit.

Single element resistance thermo-meter in Deflection Bridge



$$R_T = R_0\left(1 + \alpha T + \beta T^2 + ...\right)$$

$$R_T = R_0\left(1 + \alpha T\right)$$

$$R_T = R_{T_{\min}}(1+\alpha T)$$

$$\Rightarrow \frac{R_T}{R_{T_{\min}}} = 1+\alpha T$$

$$x = \frac{R_T}{R_{T_{\min}}} \text{ , } r = \frac{R_3}{R_2}$$

Conclusion: -

→If x is going to be increased then linearity is going to decrease.

→If r↓ = Sensitivity↓

→As the RTD is a non-linear device when the RTD is incorporated in the deflection bridge its non-linearity is compensated with the increase in its sensitivity.

Consideration

If x<<r

$$V = \frac{x}{r} - \frac{1}{r} = \frac{x-1}{r}$$

$$\frac{E_{th}}{V_S} = \left(\frac{R_T}{R_{T_{\min}}} - 1\right) \Big/ \frac{R_3}{R_2} = \frac{(R_T - R_{T_{\min}})R_2}{R_3 R_{T_{\min}}}$$

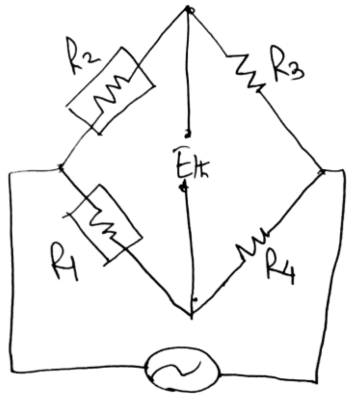
$$\Rightarrow \frac{E_{th}}{V_S} = \frac{R_2}{R_3}\left(\frac{R_T}{R_{T_{\min}}} - 1\right)$$

$$\text{For balanced condition } \frac{R_T}{R_{T_{\min}}} = 1+\alpha T$$

$$\frac{E_{th}}{V_S} = \frac{R_2}{R_3}(1+\alpha T - 1)$$

$$\Rightarrow E_{th} = \frac{R_2}{R_3}\alpha T V_S$$

Two element resistance thermometer in Deflection Bridge



→The output voltage of two element resistance Thermometer Bridge is proportional to the temperature difference between the two RTD.

$$E_{th} \propto (T_1 - T_2)$$

$$R_1 = R_0(1+\alpha T_1)$$

$$R_2 = R_0(1+\alpha T_2)$$

→In order to balance the bridge a temperature difference between the two resistance thermometers must be zero.

$$R_4/R_1 = R_3/R_2$$

$$\Rightarrow \frac{R_4}{R_0(1+\alpha T_1)} = \frac{R_3}{R_0(1+\alpha T_2)} \qquad \textit{At balanced condition } T_1 = T_2$$

$$\Rightarrow R_4 = R_3$$

$\rightarrow$ *Balanced condition for two element RTD in deflection bridge*

$$E_{th} = V_S\left[\frac{1}{1+\frac{R_4}{R_1}} - \frac{1}{1+\frac{R_3}{R_2}}\right] = V_S\left[\frac{1}{1+\frac{R_4}{R_0(1+\alpha T_1)}} - \frac{1}{1+\frac{R_3}{R_0(1+\alpha T_2)}}\right]$$

$$\Rightarrow E_{th} = V_S\left[\frac{R_0(1+\alpha T_1)}{R_0(1+\alpha T_1)+R_4} - \frac{R_0(1+\alpha T_2)}{R_0(1+\alpha T_2)+R_3}\right]$$

$$if \quad \frac{R_4}{R_0} >> 1$$

$$E_{th} = V_S \left[\frac{R_0(1+\alpha T_1)}{R_0 + R_0\alpha T_1 + R_4} - \frac{R_0(1+\alpha T_2)}{R_0 + R_0\alpha T_2 + R_3} \right]$$

$$\Rightarrow E_{th} = V_S \left[\frac{R_0(1+\alpha T_1)}{R_4} - \frac{R_0(1+\alpha T_2)}{R_4} \right]$$

$$\Rightarrow E_{th} = \frac{V_S R_0}{R_4}\left(\alpha T_1 - \alpha T_2\right) = \frac{V_S R_0 \alpha}{R_4}(T_1 - T_2)$$

$$\Rightarrow E_{th} \propto (T_1 - T_2)$$

Bridge with 4-strain gauge mounted on elastic element

Cantilever Beam (4-Strain Gauges)

$$R_1 = R_3 = R_0(1+Ge)$$

$$R_2 = R_4 = R_0(1-Ge)$$

$$E_{th} = V_S \left[\frac{1}{1+\frac{R_4}{R_1}} - \frac{1}{1+\frac{R_3}{R_2}} \right]$$

$$= V_S \left[\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right]$$

$$\Rightarrow E_{th} = V_S \left[\frac{R_0(1+Ge)}{R_0(1+Ge) + R_0(1-Ge)} - \frac{R_0(1-Ge)}{R_0(1-Ge) + R_0(1+Ge)} \right]$$

$$\Rightarrow E_{th} = V_S \left[\frac{1+Ge}{2} - \frac{1-Ge}{2} \right]$$

$$\Rightarrow E_{th} = V_S . Ge$$

Pillar Load Cell (4-Strain Gauges)

$$R_1 = R_3 = R_0\left(1+\frac{G\nu F}{AE}\right)$$

$$R_2 = R_4 = R_0\left(1-\frac{G\nu F}{AE}\right)$$

$$E_{th} = V_S\left[\frac{1}{1+{R_4}/{R_1}} - \frac{1}{1+{R_3}/{R_2}}\right]$$

$$\Rightarrow E_{th} = V_S\left[\frac{1}{1+{(AE-GF)}/{(AE+GF)}} - \frac{1}{1+{(AE+G\nu F)}/{(AE-G\nu F)}}\right]$$

$$\Rightarrow E_{th} = V_S\left[\frac{AE+G\nu F}{2AE+GF(\nu-1)} - \frac{AE-GF}{2AE+GF(\nu-1)}\right]$$

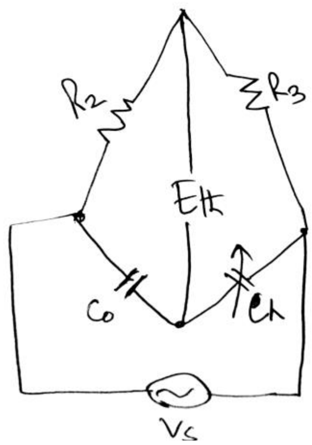
$$\Rightarrow E_{th} = V_S\left[\frac{GF(\nu+1)}{2AE+GF(\nu-1)}\right] \quad (\div\ by\ AE)$$

$$\Rightarrow E_{th} = V_S\left[\frac{\frac{GF}{AE}(\nu+1)}{2+\frac{GF}{AE}(\nu-1)}\right]$$

$$Put\ \frac{GF}{AE} << 1$$

$$\Rightarrow E_{th} = \frac{V_S}{2}\times\frac{GF}{AE}(\nu+1)$$

Design of Capacitance Deflection Bridge



$$Z_1 = \frac{1}{j\omega C_0}$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = \frac{1}{j\omega C_h}$$

$$E_{th} = V_S\left[\frac{1}{1+\frac{Z_4}{Z_1}} - \frac{1}{1+\frac{Z_3}{Z_2}}\right]$$

→For balanced condition take C_h = *Minimum*

$$E_{th} = V_S\left[\frac{1}{1+\frac{C_0}{C_R}} - \frac{1}{1+\frac{R_3}{R_2}}\right]$$

Balanced condition

$$\frac{R_4}{R_{I_{\min}}} = \frac{R_3}{R_2}$$

$$\Rightarrow \frac{C_0}{C_{h_{\min}}} = \frac{R_3}{R_2}$$

$$E_{th} = 0$$

$$\Rightarrow E_{th} = V_S\left[\frac{1}{1+\frac{R_3}{R_2}\frac{C_{h_{\min}}}{C_h}} - \frac{1}{1+\frac{R_3}{R_2}}\right]$$

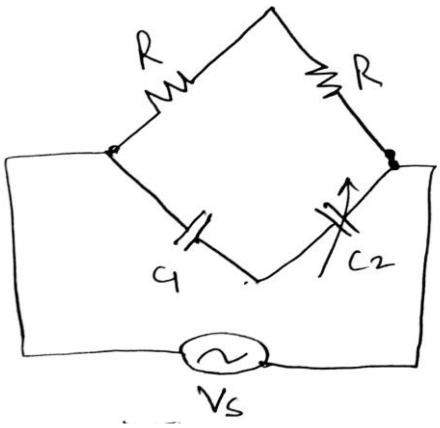
To avoid Non – linearity

$$Put\ \frac{R_3}{R_2} >> 1$$

$$\Rightarrow E_{th} = V_S\left[\frac{1}{\frac{R_3}{R_2}\times\frac{C_{h\min}}{C_h}} - \frac{1}{\frac{R_3}{R_2}}\right]$$

$$\Rightarrow E_{th} = \frac{V_S R_2}{R_3}\left[\frac{C_h}{C_{h\min}} - 1\right]$$

Capacitive differential Push-Pull displacement sensor in a Bridge element



$$Z_1 = \frac{1}{j\omega C_1}$$

$$Z_2 = Z_3 = R$$

$$Z_4 = \frac{1}{j\omega C_2}$$

$$E_{th} = V_S\left[\frac{1}{1+\frac{C_1}{C_2}} - \frac{1}{2}\right] = V_S\left[\frac{C_2}{C_2 + C_1} - \frac{1}{2}\right]$$

$$\because C_1 = \frac{\varepsilon_0\varepsilon_r A}{d + x} \quad , \qquad C_2 = \frac{\varepsilon_0\varepsilon_r A}{d - x}$$

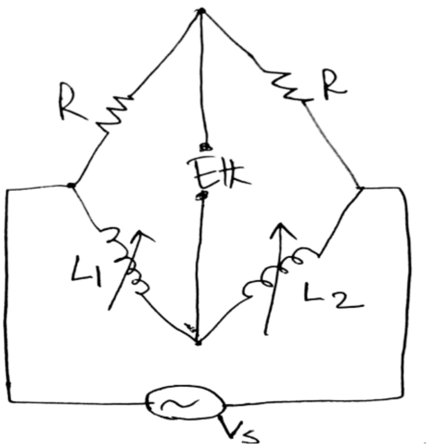
$$\Rightarrow E_{th} = V_S\left[\frac{\frac{\varepsilon_0\varepsilon_r A}{d - x}}{\frac{\varepsilon_0\varepsilon_r A}{d - x} + \frac{\varepsilon_0\varepsilon_r A}{d + x}} - \frac{1}{2}\right]$$

$$= V_S\left[\frac{\frac{1}{d - x}}{\frac{1}{d - x} + \frac{1}{d + x}} - \frac{1}{2}\right] = V_S\left[\frac{d + x}{2d} - \frac{1}{2}\right]$$

$$\Rightarrow E_{th} = \frac{V_S}{2}\frac{x}{d}$$

Conclusion: - Thevenin voltage is proportional to the displacement and if we reverse Z_1, Z_2, Z_3 and Z_4 then it is going to become non-linear. This is used for increasing the sensitivity.

Inductive differential Push-Pull displacement sensor in a Bridge element



$$Z_1 = j\omega L_1$$

$$Z_2 = Z_3 = R$$

$$Z_4 = j\omega L_2$$

$$E_{th} = V_S\left[\frac{1}{1+\frac{Z_4}{Z_1}} - \frac{1}{1+\frac{Z_3}{Z_2}}\right]$$

$$= V_S\left[\frac{1}{1+\frac{L_2}{L_1}} - \frac{1}{2}\right]$$

$$L_1 = \frac{L_0}{1+\alpha(a-x)}$$

$$L_2 = \frac{L_0}{1+\alpha(a+x)}$$

$$\Rightarrow E_{th} = V_S\left[\frac{1}{1+\frac{1+\alpha(a-x)}{1+\alpha(a+x)}} - \frac{1}{2}\right]$$

$$\Rightarrow E_{th} = V_S\left[\frac{1+\alpha(a+x)}{1+\alpha(a+x)+1+\alpha(a-x)} - \frac{1}{2}\right]$$

$$= V_S\left[\frac{1+\alpha(a+x)}{2(1+\alpha a)} - \frac{1}{2}\right]$$

$$= V_S\left[\frac{1+\alpha(a+x)-1-\alpha a}{2(1+\alpha a)}\right]$$

$$\Rightarrow E_{th} = \frac{V_S}{2}\left[\frac{\alpha x}{1+\alpha a}\right]$$

AC Carrier System

Problem in conditioning low level DC signal from Sensor Output is:

External Noise Interference: -

a) It includes the Ac Power Circuit which produce the interference signal in the measurement system

b) It may be due to external power distribution line and heavy-rotating machines like generator and turbines.

c) DC power circuits.

d) Switching DC and AC power circuits when turbine and motor are taken off line or brought back on line this causes sudden large changes in power which can produce corresponding transient in measurement system.

e) *Corona effect: -*

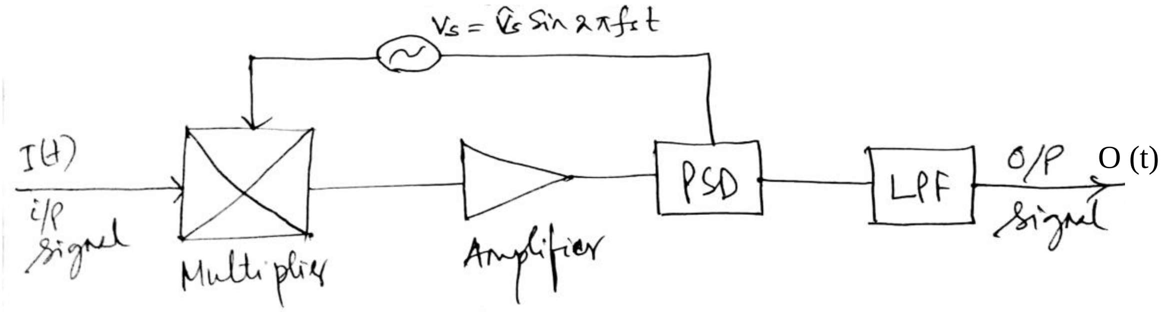
→ The air in the vicinity of high voltage power circuits become ionised and a corona discharge results.

→ The corona discharge from DC circuit results in random noise in measurement circuit and the corona discharge from the AC circuit result in sinusoidal at power frequency.

f) *Florescent Light:-*

→ The radio frequency transmitter, welding equipment and electrical discharge in the industry can produce the radio frequency interference at frequency of several Mega Hertz.

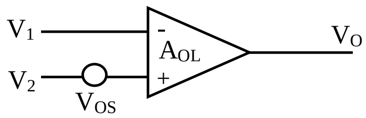
Block Diagram of AC carrier system:



PSD→ Phase Sensitive Demodulator

Amplifier →Increases the strength of signal to avoid amplifier drift.

→Amplifier Drift



$V_O = (V_2 - V_1) A_{OL} + V_{OS}. A_{OL}$

V_{OS}→ Input offset voltage = It is changed with temperature of environment and +ve, -ve power supply (+V_{CC} and -V_{EE}) so there is a drift in output voltage V_O, This is called Amplifier Drift.

$$e = \hat{e} \sin 2\pi f_i t$$

→Taking example of 4 strain gauge Deflection Bridge

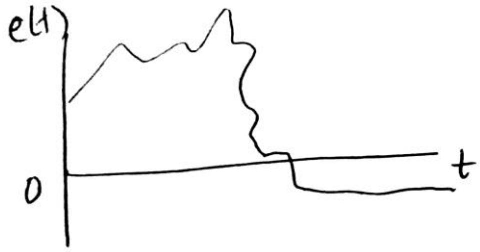
$$E_{th} = V_S G e$$

→The input signal to the AC carrier system is the AC power Supply and strain produced in the Strain gauge.

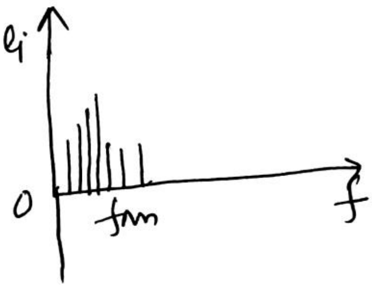
→In AC carrier system the supply voltage is $V_S = \hat{V}_S \sin 2\pi f_S t$

→The strain changed is due to force applied in cantilever will be result in a variation of strain or, damped sine wave can be expressed as a sum of many sine wave.

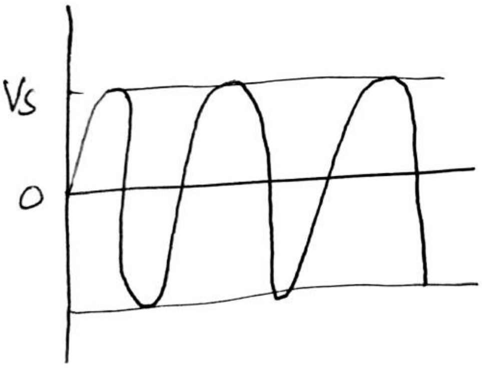
$$e(t) = \sum_{i=1}^{m} \hat{e} \sin 2\pi\, f_i t$$



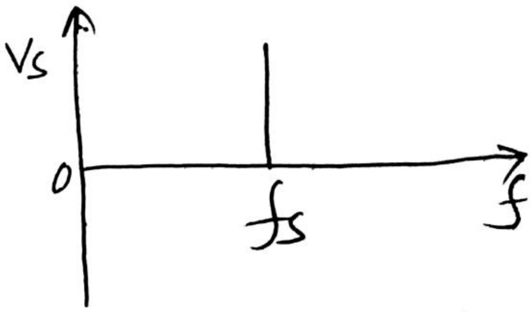
Waveform of measured signal



frequency spectrum



Waveform of Supply Voltage



Frequency Spectrum of Supply Voltage

$$E_{th} = \sum_{i=1}^{m} G\hat{V}_S \sin 2\pi f_S t . \hat{e} \sin 2\pi\, f_i t$$

$$= \sum_{i=1}^{m} \left(G\hat{V}_S \hat{e} \right) \sin 2\pi\, f_S t . \sin 2\pi\, f_i t$$

$$\Rightarrow E_{th} = \sum_{i=1}^{m} \left(\hat{V}_i \right) \sin 2\pi\, f_S t . \sin 2\pi\, f_i t$$

Where, $\hat{V}_i$ = Voltage of ith of component of strain if $E_{th}=V_i$

$$V_i = \widehat{V_i} \sin 2\pi\, f_S t . \sin 2\pi\, f_i \qquad \rightarrow \text{ Output Equation of Multiplier}$$

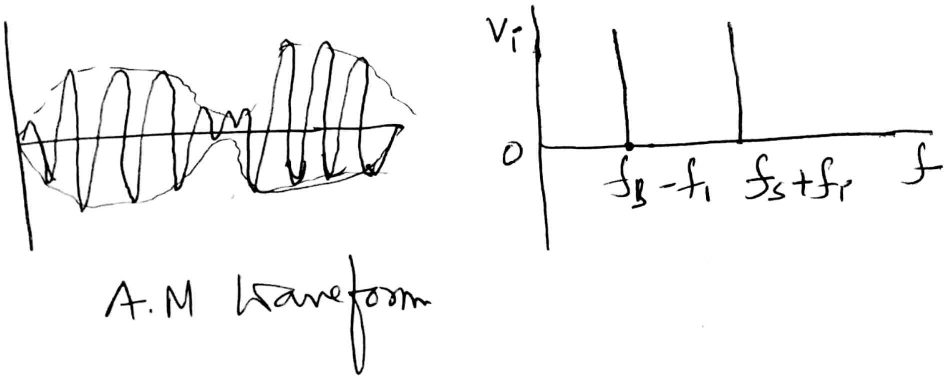
$$V_i = \frac{\widehat{V_i}}{2} \cos 2\pi \left(f_S - f_i \right) t - \frac{\widehat{V_i}}{2} \cos 2\pi \left(f_S + f_i \right) t$$

$$\underset{LSB}{\downarrow} \qquad\qquad \underset{USB}{\downarrow}$$

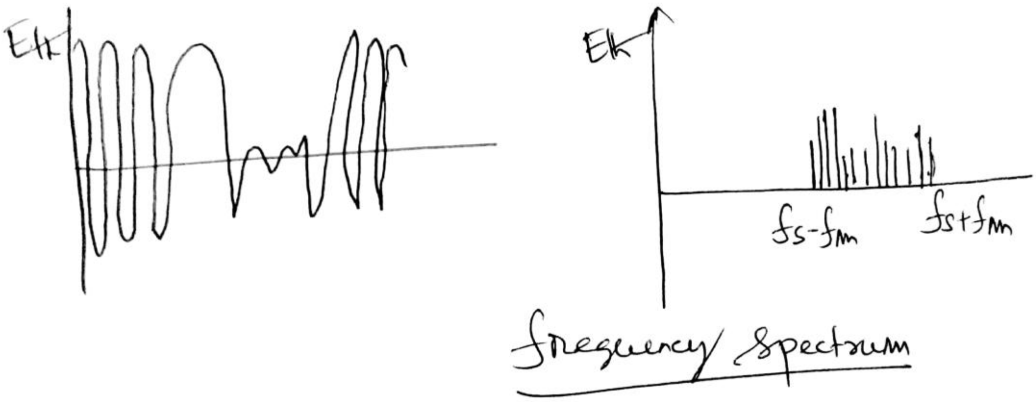
Where, f_S = Carrier Frequency

f_i = input frequency

→The amplitude modulated signal has only side band frequency and no carrier frequency. This type of modulation is called double side band supressed carrier.

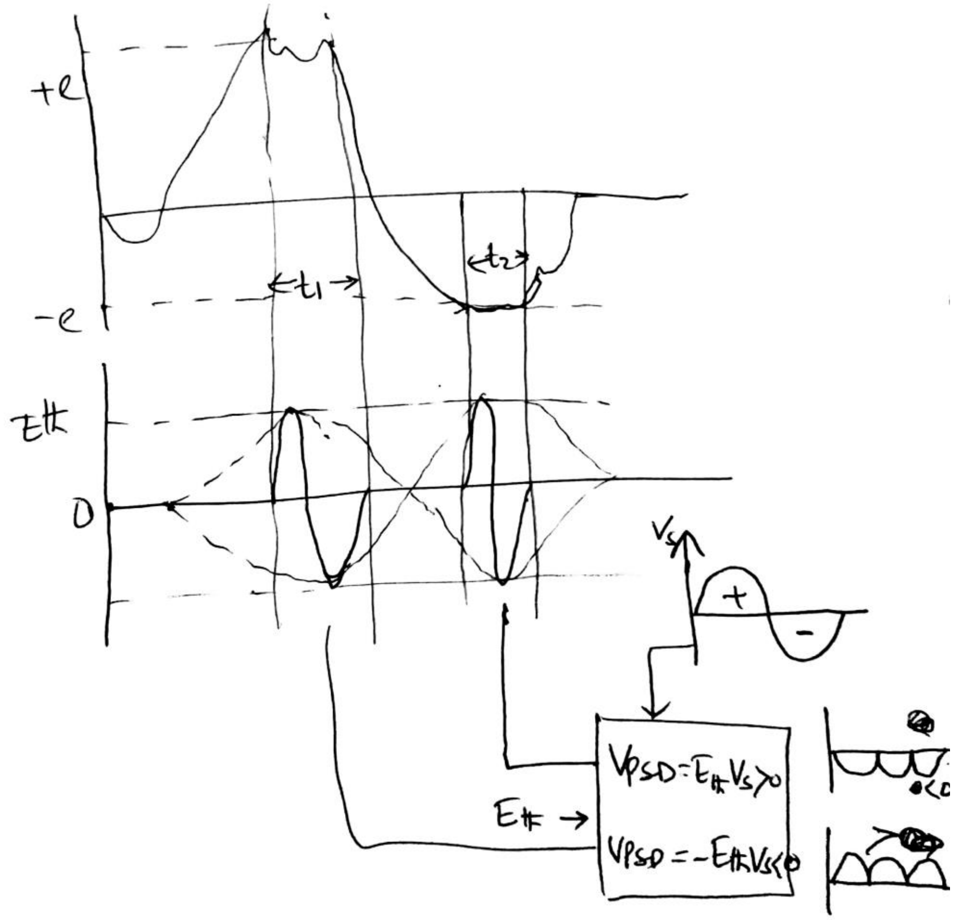


→The drift voltage that coming in to existence at the output of the bridge element due to the drift in supply voltage V_S is multiplied by the use of AC amplifier and AC carrier system.



→The AC amplifier in the AC carrier system amplifying the low level output signal of bridge element.

PSD



PSD→ Amplifier with unit gain i.e. ±1

→It is an amplifier with unity gain, the output which depends on the polarity of supply

→Voltage V_S when the force applied to the beam is downward the strain gauge 1 and 3 will experience a tensile strain +e, so the input PSD is given by:

$$E_{th} = +Ge\widehat{V}_S \sin 2\pi f_S t$$

→When the force is applied upward the input to the PSD is

$$E_{th} = -Ge\widehat{V}_S \sin 2\pi f_S t$$

$$\begin{aligned} \Rightarrow E_{th} &= -Ge\widehat{V}_S \sin 2\pi f_S t \\ &= Ge\widehat{V}_S \sin\left(2\pi f_S t + 180^\circ\right) \end{aligned}$$

→The mid-band frequency range of AC amplifier should include the frequency of LSB and USB of bridge output signal for better amplification.